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Testing for Critical Thinking in Physics

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THERE seems to be a general agreement among those who think about the objectives of science education that ability in the methodology of science is one of the most desirable outcomes of instruction in science.[1–4] For example, Bernal has said that one of the two purposes of training in science is "to give a technical understanding of scientific method sufficient to be applicable to the problems which the citizen has to face in his individual and his social life."[1]

Although there is some disagreement as to whether a satisfactory definition of scientific method can be given,[5] the weight of evidence seems to indicate that there exists a set of procedures or behaviors which nearly all scientists will agree on as constituting the elements of scientific method. A recent description of scientific method has been given by Keeslar.[6] As this description makes plain, the scientific method is a problem-solving procedure, a method for discovering that which was not known before. It is the stock in trade of the professional creative scientist. Because it is a creative process which requires great ingenuity, it has been used in the past, and will probably continue to be used in the future, by relatively few people. Since most of our students, especially in the general physics courses, are not intended for creative scientific work, we can hardly expect them to become experts in scientific method, as this is usually understood.

There are, however, included in scientific method, certain procedures or behaviors which help one to avoid error, Ability in these procedures is of importance to everyone, and should be stressed in science teaching. Thus Dewey, in discussing the aims of science education, says: "The end of science teaching is to make us aware of what constitutes the more effective use of mind, of intelligence; to give us a working sense of the real nature of knowledge, of sound knowledge as distinct from mere guesswork, opinion, dogmatic belief, or whatever.

"...An ability to detect the genuine in our beliefs and ideas, the ability to control one's mind to its own best working, is a very precious thing. Hence the rightful place of science in education is a fundamental one, and it is correspondingly important to see to it that methods of teaching are such as to fulfill its true purpose."[3]

Dewey is not describing scientific method but rather something like that which was called by

* Now at Harvard University Graduate School of Education, Cambridge 38, Massachusetts.
[1] J. D. Bernal, "Science teaching in general education," *Science Education* 29, 233 (1945).
[2] R. I. Rush, "Determining and implementing objectives for a general course in the physical sciences," *J. General Education* 2, 138 (1948).
[3] John Dewey, "Method in science teaching," *Science Education* 29, 120 (1945).
[4] O. Keeslar, "A survey of research studies dealing with the elements of scientific method as objectives of instruction in science," *Science Education* 29, 212 (1945).
[5] H. Kruglak, "Delusion of the scientific method," *Am. J. Physics* 17, 23 (1949).
[6] O. Keeslar, "The elements of scientific method," *Science Education* 29, 273 (1945).

Downing the "safeguards of scientific thinking"[7] and is very often called ***critical thinking***. There can be no question that what Dewey describes is a valuable outcome of training in physics both for the future physical scientist[8] and for those who would enter other fields.

Ability in critical thinking will not grow automatically; the teaching process must be consciously directed toward that end. There must be some attention paid to the cultivation of this ability just as there is attention paid to the acquisition of information and the development of the ability to apply principles of science to the solution of problems. Since tests are needed to guide instruction, motivate learning, and evaluate outcomes, it follows that tests of critical thinking are needed.

Before any test of critical thinking can be constructed, indeed, before an orderly attempt can be made to teach for the acquisition of ability in critical thinking, the concept has to be made more precise. We have to know just what we mean by critical thinking. It will prove most useful to give an operational definition to the phrase "critical thinking," since a concept is much more tractable when it has been so defined.[9] In order to define critical thinking operationally, we may first observe that a person who has ability in critical thinking must exhibit some behavior different from that of a person who lacks this ability. Certain behaviors, then, constitute the elements of critical thinking, and a person is said to have ability in critical thinking insofar as he exhibits these behaviors. This set of behaviors constitutes an operational definition of critical thinking, or at least the first step toward such a definition.

Once the concept is carefully delimited, test items can be constructed which are designed to elicit the relevant behaviors. The extent to which the ability is possessed by an individual can then be inferred from his responses to a set of these items. If a useful definition of the concept were not agreed upon in advance, there would be no criterion of item validity. That is, we could not tell whether an item was testing for critical thinking, or for something else, since we could not decide what was critical thinking. Each test which purported to measure critical thinking would be a separate definition of this behavior; there would be as many definitions as there were tests in the field.

Critical thinking as such is an abstraction and can have a concrete meaning only when applied to some subject matter. When the subject matter is in the field of chemistry, we have critical thinking in chemistry, etc. Therefore, in order to complete the definition of critical thinking which follows, one must think of the behaviors which constitute the elements of critical thinking as applying to some branch of empirical science. This definition is to be regarded as tentative and subject to modification according to the needs of those who may use it.

Definition of Critical Thinking

A person carries on critical thinking when he exhibits the following behaviors:

1. Differentiates between authoritative and non-authoritative (reliable and less reliable) sources of information.
2. Criticizes faulty deductive reasoning.
3. (a) Differentiates between statements that describe observations, i.e., "facts," statements which are hypotheses about facts, and statements that introduce new words.[8]
   (b) Recognizes meaningless statements, e.g., statements which are not definitions, are not verifiable by observations or do not have implications verifiable by observations, and are not mathematical or logical propositions.
   (c) Recognizes evidence of personal bias in statements.
4. (a) Draws valid inferences from graphs, tabular data, expository material and other given information.
   (b) Recognizes what assumptions are to be maintained in drawing inferences from data.[6]
5. Selects data which are pertinent to a problem.
6. Criticizes data (tabular, graphical, or other) which have been collected to aid in the solution of a problem, with respect to:
   (a) pertinency to the problem,
   (b) accuracy of the data and reliability of the method of collection,
   (c) sufficiency.
7. Criticizes inferences drawn from data by recognizing whether a supposed inference is an implica-

[7] E. R. Downing, "Elements and safeguards of scientific thinking," *Scientific Monthly* **26**, 231 (1928).

[8] P. Frank, "The place of the philosophy of science in the curriculum of the physics student," *Am. J. Physics* **15**, 209 (1947).

[9] There is a discussion of operationism in psychology by C. C. Pratt in *The logic of modern psychology* (Macmillan, ed. 1), pp. 81–120.

tion of the data, unrelated to the data, or contradicted by the data.

8. Estimates probability of an inference and criticizes given estimates of probability.
9. Selects the hypothesis, from a group of hypotheses, which most adequately explains given data.
10. Recognizes the approximate or tentative nature of hypotheses.
11. Recognizes what assumptions, beyond the data, have to be made in the formation of hypotheses.[6]
12. Criticizes hypotheses as to:
    (a) accordance with the data,
    (b) adequacy of explanation.
13. Criticizes experimental procedures as to:
    (a) pertinency of the procedure to the problem,
    (b) isolation of the experimental variable by proper controls,
    (c) accuracy of the observations,
    (d) sufficiency of the number of observations or repetitions of the experiment,
    (e) validity of the assumptions involved in setting up the experiment.
14. (a) Recognizes the existence of errors of measurement,
    (b) recognizes when the precision of measurement given is of a degree warranted by the nature of the problem,
    (c) criticizes a stated precision of measurement according to the precision of the measuring instruments used.
15. (a) Recognizes what assumptions have to be maintained in generalizations from the results of an experiment,
    (b) criticizes the validity of generalizations from the results of an experiment to new situations according to the degree of similarity of the new situation to the experimental situation.

It should be noted that the formulation of hypotheses and the devising of experimental procedures have been omitted from the definition because it was felt that these behaviors represent creative rather than critical thinking. The writer feels that it is an open question as to whether grouping these two activities in the definition would be appropriate. Of course, there is a great deal of overlap between those behaviors which may be termed critical and those best described as creative. The decision as to where to draw the line has been based on pragmatic considerations. Thus, the drawing of valid inferences from given data (element 4(a) of the definition) has been included as a part of critical thinking because it may be readily taught and tested in conjunction with the other behaviors.

The construction of physics items which will elicit the behaviors given in the definition of critical thinking is a difficult task. The natural tendency is to repeat with but slight modification the teaching and testing techniques to which one was exposed in his own schooling. Although this may generally be a good thing, it is not conducive to the development of new techniques.

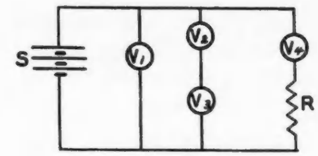

FIG. 1. Circuit diagram.

Unquestionably there is a general lack of experience in the field of testing for critical thinking, and one should not hope for too rapid progress. Attempts have been made, however, and are currently being made, to test for critical thinking in physics and in other fields; and much can be learned from the failures and successes of the pioneers in this work. In the field of physics, one example is the United States Armed Forces Institute Examination in Physics—College Level, Section III.[10] This section is labeled "Critical Thinking in Physics." Examination of this test shows, however, that the ability to respond correctly to the items depends rather on knowledge of facts or principles and application of principles to specific situations. These items are probably excellent for measuring certain types of achievement in elementary physics, but they do not test for critical thinking. Two representative items from this test are given below.

1. A flexible hose, lying on the ground, is carrying water at high speed. If a large leak develops in the right-hand side, what will be the resulting motion of the hose?
   A. It will arch toward the right.
   B. It will arch toward the left.
   C. It will twist so that the leak is up.
   D. It will twist so that the leak is down.
   E. Any of the above may happen, depending on the flexibility of the hose.

This item obviously tests only an application of the laws of motion. The difficulty of the item depends on understanding the third law of motion. Moreover, it would have been preferable to use a simpler dynamical situation to test this point. As anyone with considerable experience in handling garden hoses knows, a leak is likely to result in no motion at all, or else in a rather

[10] *U.S.A.F.I. Examination in Physics—College Level, Section III* (Educational Testing Service, Princeton, N. J., 1945).

complicated and unpredictable motion. Therefore, the item as it stands may reward the student who applies the principle uncritically and penalize the one who has actual knowledge.

2. In the circuit shown in Fig. 1, V_1, V_2, V_3, and V_4 are accurate voltmeters: R is a resistance: S is a battery. Which pair of voltmeters are sure to have the same reading?
   A. V_1 and V_2.
   B. V_2 and V_4.
   C. V_1 and V_4.
   D. V_2 and V_3.
   E. None of these.

It seems to the writer that responding correctly to this item requires mainly knowledge of the construction of a voltmeter and of the behavior of voltmeters in electric circuits. The elements of critical thinking are not involved to any great extent. In addition, it should be pointed out that the use of the word "sure" in the item makes the correct answer fairly obvious.

The following set of items prepared by the writer is presented as an indication of how one may test for some of the elements of critical thinking as applied to physics.

Items 3 through 5 are based on the following information: "Microscopic particles of nickel have been observed to move in the direction of the lines of force in a vertical homogeneous magnetic field when illuminated by a strong beam of light. The velocity of the particles is a function of the field strength. The direction of motion reverses with a reversal of the magnetic field. For low field intensities, the velocity is directly proportional to the field strength. For higher intensities, the velocity either approaches a limiting value or reaches a maximum with increasing field and then decreases. Increase of field strength may even reverse the sense of direction of the motion." (G. Kane, "Magnetic saturation of microscopic particles," *Physical Rev.* **67**, 63 (1945).)

These observations are offered in support of the theory that magnetic ions exist. The data are explained on the basis of the hypothesis that the particles of nickel carry single magnetic poles, i.e., they are "magnetic ions."

3. It may be inferred from the given information that the magnetic field mentioned is most probably created by
   A. an Alnico horseshoe magnet,
   B. an electromagnet,
   C. a strong beam of light,
   D. particles of nickel,
   E. a cobalt magnet.
4. Of the following, which fact is *least* adequately explained by the hypothesis?
   A. The reversal of the direction of motion with increasing field strength.
   B. The movement of the particles along the magnetic field instead of at right angles to it.
   C. The dependence of the velocity of the particles on the field strength.
   D. Reversal of the direction of motion with reversal of the magnetic field.
5. Which assumption, unsupported by the data, is implicit in drawing the conclusion that the particles of nickel carry single magnetic poles?
   A. Nickel is a ferromagnetic substance.
   B. The particles of nickel are not propelled by radiation pressure.
   C. The resistance to the motion of the particles is directly proportional to the velocity of the particles.
   D. The electric field is zero in the direction of the magnetic field.

It should be noted that the items given above do not test for critical thinking apart from physical content. They would be useful therefore, only for students with sufficient background in electricity and magnetism. Apparently it is a practical impossibility to present enough information with a set of items to make the difficulty of the items entirely independent of subject-matter knowledge. The best that can be done is to make the items suitable for a group with some particular level of subject-matter competence.

The American Chemical Society's Committee on Examinations and Tests has come closer to testing for what has been defined as critical thinking, as applied to chemistry, in Part IV of its *General Chemistry Test*.[11] This part is labeled "Scientific Method." Several items from this test are given here to illustrate how it is possible to test for some of the behaviors given in the definition of critical thinking.

6. Free sulfur may act as either an oxidizing or reducing agent.

A. What is the best experimental evidence for this statement?	1. Sulfur forms sulfur dioxide as well as copper sulfide.
B. What is the best theoretical explanation of this statement?	2. Sulfur is a nonmetal.
	3. Sulfur forms both organic and inorganic compounds.
	4. With six electrons in its outer shell the sulfur atom can either form a negative ion by gaining two electrons

[11] *A.C.S. Cooperative Chemistry Test for College Students, Form 1948* (Educational Testing Service, Princeton, N. J.).

from other elements or share its electrons with other atoms.
5. Sulfur is soluble in carbon disulfide and also in alcohol.

The difficulty in item 6-A depends chiefly on the recognition of a statement which describes an observation rather than a hypothesis. The difficulty in item 6-B is similar, requiring the opposite distinction.

7. When sodium hydroxide solution is added to magnesium sulfate solution, a white precipitate of magnesium hydroxide is obtained. When sodium hydroxide solution is added to an "unknown" solution, a white precipitate is obtained. In order to conclude that the unknown solution contains magnesium ion, it must be assumed that
   1. NaOH is more soluble than $Mg(OH)_2$,
   2. Na_2SO_4 is soluble in water,
   3. $Mg(OH)_2$ is insoluble in water,
   4. NaOH forms no white precipitate with any other ion except Mg^{++},
   5. Zn^{++}, which forms white $Zn(OH)_2$, is not present in the unknown.

Little knowledge of chemistry is needed to answer the item above. Rather, it requires that the student recognize what assumption has to be maintained in drawing an inference from the given data (element 4(b) of the definition).

8. Consider the following assumptions, from the original statement of Dalton's atomic theory, in answering items A and B.
   1. The ultimate particles of matter are the atoms which are indivisible and indestructible.
   2. All the atoms of a given element are alike in all respects.
   3. The atoms of different elements differ in one or more properties.
   4. Compounds are formed by combination of different kinds of atoms.

   A. Which of the above assumptions must be revised or discarded because of the existence of isotopes?
   1. 1
   2. 2
   3. 3
   4. 4
   5. None.

   B. Which of the above assumptions must be revised or discarded because of the invention of the atomic bomb using plutonium?
   1. 1
   2. 2
   3. 3
   4. 4
   5. None.

Although some knowledge is required to respond correctly to the preceding items, there is also need for criticism of a hypothesis in accordance with certain data.

9. The table below gives the vapor pressure in millimeters of mercury of three liquids at various temperatures. Use the data given in the table in judging the statements made in items A through D.

	0°C	20°C	50°C	80°C	100°C
Benzene	25.3	75.6	271.4	751.9	1340.0
Turpentine	2.1	4.4	17.0	61.3	131.1
Water	4.6	17.6	92.0	354.9	760.0

Consider each statement (A, B, C, D, below) and select the one of the following five choices that best indicates the degree of correctness of the statement:
1. The statement is *true.*
2. The statement is *probably true;* additional data would be needed for a final decision.
3. It is *impossible to judge* the statement because the data are insufficient.
4. The statement is *probably false;* additional data would be needed for a final decision.
5. The statement is *false.*

A. The normal boiling point of benzene is lower than the normal boiling point of turpentine.
B. Water will not evaporate below a temperature of 100°C.
C. The normal boiling point of turpentine lies between 400°C and 500°C.
D. The normal boiling point of benzene at standard pressure is 82°C.

These items require criticism of inferences drawn from data. The kind of critical thinking required, however, is of a rather low order compared to the knowledge of principles needed to understand the data.

Other attempts to test critical thinking in various fields have been made, notably the *Interpretation of Data* tests of the Progressive Education Association,[12] certain Cooperative Tests[13–15] published during the thirties, and others, many of which are probably unfamiliar to the writer. Most of the items in the previous tests in this area require principally the criticism of inferences drawn from given data, although

[12] *Progressive Education Association Tests 2.51 and 2.52, Interpretation of Data* (Educational Testing Service, Princeton, N. J., 1940).
[13] *Cooperative Botany Test, Part II, 1934–1936* (*out of print*) (Educational Testing Service, Princeton, N. J.).
[14] *Cooperative Zoology Test, Part II, 1934–1936* (*out of print*) (Educational Testing Service, Princeton, N. J.).
[15] *Cooperative Chemistry Test, Part II, 1934–1936* (*out of print*) (Educational Testing Service, Princeton, N. J.).

items in a test by Engelhart and Lewis[16] test the relevance of data to various hypotheses.

A good deal of work along these lines has been done at certain institutions for local use rather than for publication or use in large-scale testing programs. Nedelsky, at the University of Chicago, for example, has put together an especially valuable manual of examination questions, much of which is in this general field.[17]

[16] Engelhart and Lewis, "An attempt to measure scientific thinking," *Educational and Psychological Measurement* **1**, 289 (1941).

[17] Leo Nedelsky, *Testing for specified objectives of physics teaching* (University of Chicago; distributed by the author).

Taking into consideration previous experience in the field, we may say that if we recognize the need for emphasizing critical thinking in our physics courses, it should not prove an insuperable task to construct a useful test in critical thinking in physics. Although it would be difficult or practically impossible for a busy teacher to devote the necessary time and energy to constructing the large number of elementary items needed for the test of this type, a group of reasonable size working in cooperation should find it possible to do so. The work of such groups has been very successful in the past, and it seems that a concerted, well-thought-out attack on this problem will result in a useful test.

The "Block-and-Gap" Scheme for Physics Courses*

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GENERAL education is in the air. "General Education Science Courses" is a new, and fashionable, term. Makers of new courses are reducing aims to schemes; proud owners of old courses are reviewing their gains. While, from the nature of things, there is agreement on the general purpose of such courses, no common set of aims emerges, and no single course-scheme is welcomed as best. In fact, there is a vitality of growth in the diversity of aims and schemes. Each of us concerned with such a course is eager to compare notes with others. However, uncertainties of terminology hamper such discussions, and lead to unnecessary disagreements. For example, merely mentioning "survey courses" will often start a row. Without restricting the healthy diversity, one can extract some very general requirements for science courses in general education and lay down some descriptive terminology. I shall attempt this in this paper. Such a discussion still leaves largely unrestricted the choice of subject matter and its treatment—e.g., historical approach *vs.* laboratory study. My aim is to promote discussion, not hamper it by insisting on some ideal scheme. With this aim in mind, I shall first review the intentions of the new courses, discuss their aims (inconclusively), comparing them with orthodox physics courses, and finally arrive at a general scheme, the block-and-gap course, which is likely to provide a useful framework for the new courses.

* This article is based on an invited paper read at the meeting of The American Association of Physics Teachers in January 1949, and on the introductory chapter which I wrote for *Science in General Education*, edited by Earl J. McGrath. (Published by Wm. C. Brown and Company, Dubuque, Iowa.) Considerable portions of the material of that chapter are used here. The whole book contains detailed descriptions of a number of the new courses by different authors.

Need for New Science Courses

Much of the welfare of civilization, and perhaps even its fate, depends on science. Do our science courses educate students to understand this dependence? Scientists have a characteristic way of thinking and planning and working, which we call the scientific attitude or scientific method or science itself, that offers intellectual resources and guidance to all students. Do our science courses send their students out delighted with that understanding of science, and ready to turn it in new directions? Can governors and administrators who have taken our science courses confer intelligently with scientists on the vital problems of our age? In general, does our science teaching in school and college make its

proper contribution to general education? Even in the matter of teaching some science, are we meeting students' needs and hopes?

Children are thrilled with the idea of scientific experiments and knowledge. Many a small boy is eager to learn physics and chemistry. When we show him a plain test tube, his tongue hangs out with enthusiasm. He longs to play with the first magnet he sees. Yet a few years of science classes—including, say, some qualitative analysis or a study of magnetic-field formulas—will deaden the enthusiasm in almost all students. A few emerge still determined to be scientists—but even they usually have a strange picture of science as a sort of stamp-collection of facts, or else as a game of *getting the right answer*. For the majority, well-meant teaching has built a wall around science, a stupid antagonistic wall of ignorance and prejudice.

In general education, we need not start the training of professional scientists (that can be done much faster once the vocation is chosen); we need not try to equip everyone with a lot of scientific knowledge (that can be stored in books or left to the professionals); but we do need to give an understanding of science and its contributions to the intellectual, spiritual, and physical aspects of our lives.

Suppose we think of our own children in college concentrating in economics or languages or history, but taking some science courses as part of their general education. With what questions should we test the *success* of such courses? We should hardly be content to ask: "How many facts have they learned?" Facts are forgotten all too soon. We are more likely to ask: "Can they think scientifically? Do they understand what science is about and how scientists go about their work? Have they a friendly feeling toward science and scientists? Are they likely to read scientific books in later life with enjoyment and understanding? Could they work with scientific advisers in business or government? Could they enjoy intellectual discussions with scientists?" In asking such questions as parents and educators, we betray some of our educational aims. We should not agree on all our aims, but I think most of us have in common a number of aims and hopes which form a cogent group, demanding quite a different kind of science course from the orthodox ones now given in many colleges.

The Case for (and Against) Orthodox Courses

The orthodox course—easily named, but really a variable character hard to describe—is useful as the beginning of professional scientific training; but in this discussion it is one of the villains of the piece. For many years colleges all over the world have offered formal elementary science courses to their students, to science specialists and others alike. In most cases, these have grown to be courses in a single science with almost standardized content.[1] Emphasis tends to be on content rather than on ideas or scientific method or even thorough understanding; and the courses are intended to provide a sound foundation for more specialized work in their science. Many nonscientists take such courses, under university rules enforcing "broad general programs" or through their own interests and choice. Colleges have defended the use of such courses for general education on grounds such as the following (I have added a parenthentical chorus of criticism):

1. A thorough grounding in science does show the nature of science. (However genuine this aim, I doubt whether real students find that orthodox courses do this for them. The topics seem to be crowded and unfinished. The teacher seldom has time to point the moral.)

2. Acquaintance with the main facts of a science is itself a valuable part of education for civilized life. (Facts are soon forgotten or muddled, particularly when delivered with authority and speed. If "education is what is left after what you learn has been forgotten," the providing of fact-content should not be the sole aim in science courses for general education.)

3. The discipline of thorough study, including learning material that is boring or difficult, is valuable in itself. (Under criticism from psychologists, this kind of argument has lost favor in the field of classics. In science it is likely to lose favor for the same reason. Also, it is likely to be crowded out by other aims.)

4. Work in science gives training in scientific method—that is, it makes people more scientific

[1] For example, nearly all physics texts intended for first-year college courses have almost identical content and much the same order of topics as each other. Each author seems afraid to omit what his rivals include.

—a virtue to be transferred to other studies and other activities in general life. (This gives a cogent reason for any studies which do provide such benefits. Investigations show that such transfer of training does not occur easily or in great measure. To encourage it, we need to modify our teaching, as we shall see later.)

5. A taste of science in a first college course gives some students a chance to decide they will be scientists. (This is true; but it may not be necessary to offer the samples in the form of orthodox courses. In any case, one single-science course fails to give students much width of persuasion in their choice.)

Science Courses for the General Student

In discussions of general education, the suitability of orthodox science courses has been questioned, and some colleges offer or are planning science courses which they hope will give greater benefits to the education of the general student. These new courses differ among themselves in schemes of content and teaching and even in their underlying aims. Obviously, differences of aim are likely to lead to differences of treatment. So at first thought we must settle our aims before we can give such courses. Yet a full discussion of aims would be endless, leading to unresolvable differences of opinion, tangled with educational difficulties rooted in differences of educational, social, and political ideology. Committees have succeeded in evolving statements of aims, but the resulting lists are too impractical and too long. Yet there are common elements among the aims that different people ask for in science courses for the general student, and I believe that we can make courses that foster such aims.

Let us pay little attention to grandiose aims which are beyond the scope of real science courses with flesh-and-blood teachers and students—aims such as fostering emotional balance or producing great thinkers. And let us pigeonhole aims which seem to us as physicists side issues, e.g., promoting good spelling. What are the realistic aims we might have before us in actually constructing and running a course? I think we can find common elements among the realistic aims that different people ask for in courses for the general student. So I shall give an inconclusive, unscientific discussion of aims, to show the general line of thought.

Aims of Science Courses

*(1). **Content vs. training and understanding.***—Imparting scientific knowledge is common to all science courses, with aims ranging from making healthier citizens to training better soldiers for mechanized warfare. We rely on special courses to give specialized training; but, in general education, while some acquaintance with fact comes anyway, rigorous courses in facts and principles are disappointing. Perhaps we can neglect some of the fact-material now taught; then the rest may profit from more careful teaching so that the student actually remembers more material, not less, some time after the course. Students who have a good understanding of the nature of science should be able to look up facts in books, and are likely to retain a lifelong interest in scientific reading.

So we have come to think that the more important things to aim at in science courses in general education are such things as understanding what science is about and knowing how scientists go about their work, rather than material knowledge alone. To give such understanding, we must modify the orthodox courses, paying quite a price, in terms of technical training, for the gain we hope to make. So we ask, what are the values to be sought, and how can we construct a course that is likely to give them? When we try to answer these questions, the problem of transfer appears again and again; so I shall discuss that problem before suggesting a list of aims.

*(2). **The problem of transfer of training.***—"Will students transfer training, in some skill or habit or the use of some idea, from a science course to other studies or to life in general?" This is a vital question. If the answer is "no," our new schemes must relate merely to better training inside a science, and offer little promise as a part of general education. If the answer is "yes," our hopes should be grand indeed. In earlier generations, courses in classics, history, mathematics as well as science—in fact most of higher education—claimed cultural values on the ground that their teaching would transfer to many other fields of the student's education and there be retained as part of his general culture. Educa-

tors pointed to the high levels of scholarship and culture "produced" by a thorough classical education. In this they seem to have risked some confusion between *post hoc* and *propter hoc*—we might suggest their classical scholars had the intellect and background to succeed anyway. There have been growing doubts about this hoped-for transfer. Are scientists themselves better for their studies: tidy and systematic in their general life, critical and unbiased in their general thinking?

Since early this century, experimental investigations at first said "no" to our question about transfer, then later studies showed that it can occur to some extent. It certainly does not take place as easily as educators and the general public hoped. If it did not occur at all, higher education would seem almost worthless except for special professional training. Fortunately there is some transfer—language teaching can improve intellectual skills, mathematics can give a sense of form or give training in careful argument, and so on—but only in certain favorable circumstances. In our present discussion, it is essential to know what these favorable circumstances are and to try to provide them. They seem to be:

1. There must be common ground between the field of the training and the field to which we wish it to transfer; or there must be similarity between the influencing and influenced functions. For example, if we train a student to weigh accurately in a physics laboratory, it is almost certain that this training will transfer to another physics laboratory and he will weigh the more accurately there; it is moderately certain that he will carry his good training to a chemistry laboratory; much less likely that he will carry it to any weighing he does in his own kitchen or in his business; and it is very unlikely that the training in accuracy will reappear as a habit of being accurate in other activities. Another example: training in argument learned in geometry is likely to be transferred to later geometrical studies, not very likely to be transferred to work in physics, unlikely to help the student to think critically about arguments in newspaper advertisements, and very unlikely to make him a better economist. (We can lessen the gloomy doubts expressed in these examples by attending to the other conditions 2 and 3.)

2. Conscious efforts toward transfer can help. We can make transfer more likely by making a student aware of his gains in one field and pointing out their applicability to other fields. We should exhort him to transfer.

3. An almost essential lubricant for the process of transfer is the emotional attachment (or "sentiment") the student develops—the extent to which he associates feelings of enjoyment, interest, inspiration with his studies. The more he enjoys his science and is inspired by its skills and methods, the more he likes discussing its philosophy, the more likely he is to retain and generalize the teaching. Thus, reverting to our examples, a student who develops a *delight* in accurate weighing, making accuracy almost a minor ideal, may well carry the techniques and attitude of seeking accuracy far and wide in his activities, particularly if he has been made aware of the possibility and value of this wide transfer. The student who develops skill in geometrical argument *and* feels inspired by the method may well become the clearer lawyer or cleverer economist by the transfer of some of that training.

4. It has been suggested that ease and amount of transfer increase with increasing general intelligence. This seems reasonable in the light of the other requirements. If this is true, the brightest students should profit most from courses in general education.

(*3*). ***Discussion of some aims.***—With the difficulties of transfer in mind, we can now review some of the benefits aimed at or claimed in science courses. We can see that an orthodox course full of information, enlivened by demonstration and exhortation, may fail to make any great contribution to general education, and may even fail to educate students in science. In planning new science courses we are offered a variety of aims. I shall list some of these, with comments.

1. Teaching scientific method(s). The course should teach scientific method, that is, show how scientists attack a problem: gathering data, sorting data, making and testing hypotheses, suggesting and trying further experiments, until theory and experimental knowledge are built into the structure of a science.

I think there are two kinds of aim here: (a)

training in scientific investigation, both in science and outside and (b) showing the student what science is like. The former training would be a fine part of a student's education if we could really equip him to use it, but it is doubtful whether we can train people to behave much more scientifically in their general life. Also, the scientific method—if there *is* just one—may not be so wisely applicable to other studies such as the social sciences. Perhaps the furthest we can hope to go is to encourage a scientific attitude and critical thinking in general life.

In aim (b), showing what science is like, we can go much further. Orthodox courses try to do this by imparting knowledge of facts thoroughly, but not only do they tend to crowd out the teaching of method and the showing of the nature of science; they also give a mistaken picture of science itself by making it seem to be a body of fact provided mysteriously or drawn from sources known to only a few. However, science courses *can* be made to show the methods and growth and nature of science, perhaps by historical treatment, perhaps by other methods. But in all cases there must be time for students and teachers to discuss these matters, which should be presented with active attention to their part in the student's education. In a way, the patient needs to be reminded that he is taking good medicine and that if he cooperates he may expect great benefits.

In the teaching of scientific methods we find encouraging critical thinking as one of the aims. Here we may hope for some transfer, but only if we plan for it. We may use the course as ground in which to grow habits of critical thinking. Scientific work involves great care to avoid bias, so it is easy for the student to see the need for logical, unbiased thinking in science. However, we should not assume that mere contact with science which is so critical will make the student think critically. I think this is an important aim, not easy to achieve—often not achieved in the measure its sponsors claim or hope—to be sought by giving students time to discuss scientific material critically and creatively. Humanists would claim that their fields provide good opportunity too—for example, courses in logic, philosophy, semantics, poetry, history. However, like the scientists, they have spoiled their chances by choking their courses with too much material and they have soured the customers by their preoccupation in examinations with the "detestable testables" such as spelling and dates. Scientists feel that their field provides easy, clear-cut material for this teaching.

2. *Giving creative work.* There is a tendency in modern fact-jammed education to starve students of real creative activity. Many a graduate in science emerges with no memory of creative experience in college. Yet without such experience, civilized man is a dull fellow and general education has little chance to make him better. Students can do creative work in science if they are given time, opportunity, and encouragement instead of being told, "That was known twenty years ago," or being instructed, "Do this problem properly, the way the book tells you." Laboratory work can provide a sense of real creative work for some. Reading, with some free choice, can give it to others. For others, essays and discussions can provide it, but these should deal with thought-provoking questions rather than routine problems. With our interest turned toward it, we can give students a sense of creative activity.

3. *Cultivating general abilities and training the mind.* This sounds like an educational catch-all, but it is an expression of our general hopes for transfer from science. These are the aims I consider unrealistic, for which there is neither much hope of transfer nor much opportunity for direct treatment in a science course. Yet most course-planners mention some such aims. I suggest all teachers and planners should review these general aims before they start, then place them on a storage shelf, marked IMPRACTICAL, where they may remain as laudable ideals.

4. *Teaching basic principles.* "Understanding of the basic principles of the physical and biological world, their implications for human welfare and their influence on the development of thought and institutions" would seem, at first reading, to be just the kind of aim many of us have in mind. But can we make the student understand the basic principles? If so, will he be able to see their implications beyond science? There seems to be a hint here of a set of indispensable topics, the basic principles, making too weighty a list. A thorough study of basic principles might concentrate attention on subject

matter to the exclusion of teaching of method and ideas. Many of us doubt if we can—or even should—teach *the* basic principles and their implications. A choice of a *few* basic principles might be wiser, giving time for other matters. The particular choice may not be very important, provided this restricted choice leaves time to study the development and interrelations of the chosen topics.

5. *Showing what science is like, what scientific procedure is like, and what scientists are like.* These, I think, are the most real and probably the most important things we expect from a science course. Our courses should mediate between the scientist and the layman, between a classical culture and a scientific civilization. They cannot do this just by pouring in scientific information or even formal training. What is needed is a sympathetic understanding of science and the way scientific work is done. To make this understanding a lasting part of people's culture is a huge task. In a year's course we can give only glimpses of it.

If the student is to retain such valuable knowledge and ideas, we must give him opportunity and time to make them his own, not just have them thrown at him by a provident teacher. Some students will need to learn what scientific work is like by *doing*, by experimenting independently in the laboratory, and thinking about the meaning of their experiments. Others need not be restricted to this excellent but very slow way of learning; they can learn by studying the work of the great men of science, watching its growth in the framework of science against the background of history. Others can learn from direct teaching of science—science as it now is, with some attention to the way it has grown to its present state. Class groups with many students probably need a mixture of these approaches.

Whatever examples of scientific work are chosen, they should be studied thoroughly so that students can see the way the work was done and appreciate the methods used, so that they do not merely learn the results (for examina-

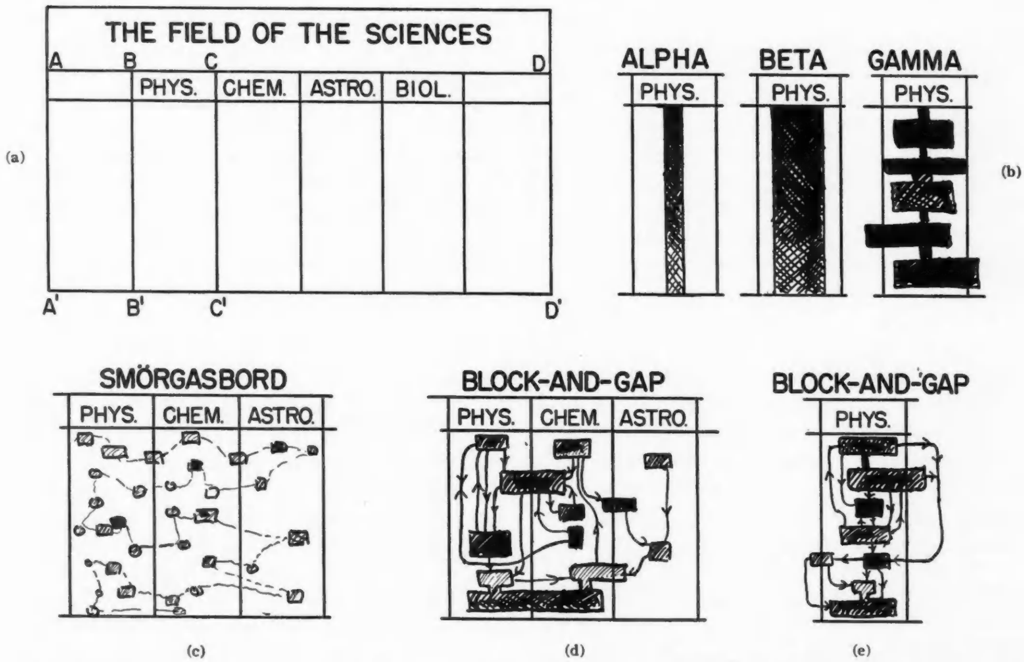

FIG. 1. Quantity and arrangement of subject matter in college science courses: (a) The field of scientific knowledge, divided into separate sciences; (b) orthodox elementary physics courses; (c) a "survey" course in physical science; (d) a block-and-gap course in physical science; (e) a block-and-gap course in physics.

tions) but see how those results take their place in the framework we call science. In particular, they should see how theory and experiment are related in each example studied. They should see how both the work and the results are related to the social and intellectual life of the times when the work was done, and perhaps see how they affected the life of later generations. Such general understanding of science needs specific examples; but these cannot just be presented as glimpses. The student must understand the facts well, then think about their meaning and relationships. The teacher must help him to turn his attention in different directions, and must encourage him to generalize the understanding he is gaining.

To understand *what science is like* is a big requirement. We have only to look at the popular misconceptions of science—in a civilized world where science has been taught to many for generations—to realize that these cannot be due to unsuitable teaching alone. To show *what scientific work is like* is easier, because separate examples can build such an understanding and the student can see more easily what kind of thing he is learning. Perhaps easiest of all is to show *what scientists are like*, not in their personal characters[2] but as thinkers and workers. Students should see how some of the great scientists approached problems, how they worked, and what their results meant to them and other scientists. They can learn the ruthless truthfulness with which scientists try to base their work on experiment. They can see experiment and theory playing their complementary roles in the hands of real scientists. They can learn how theories (or conceptual schemes, or even just "ideas") are neither wild guesses nor true pictures, but wise imaginative complexes of ideas and knowledge which coordinate thinking and promote further knowledge. If students can learn such things as these, they will form a new generation of laymen able to appreciate scientists and to work with scientist neighbors with delight and understanding.

[2] Some teachers suggest that the actual lives and characters of great men of science form an inspiring example for students, but many of us disagree. I would dread to see a government scientific project run by Tycho Brahe, Galileo, and Hooke. Personal details are imported by professional historians (under a claim of accuracy). I feel such details are unwanted here except where they make the nature of the scientist's work clearer.

Looking back over this list of overlapping aims and hopes, we see we shall want to construct our courses in ways likely to promote transfer. I shall not carry this discussion of aims further. Each of us should make his own list and decide his own order of importance. In spite of differences most of us will agree that our aims for such science courses go far beyond fact-content or even scientific principles and include much more general understandings.

Quantity and Arrangement of Subject Matter

Our discussion of aims suggests that while details of content are not very important, the *quantity* of content and the general structure of topics are matters of great importance in the new courses. Accomplishment of most of the aims requires time. There must be time for student discussion, for careful reading, for historical analysis, for arguments and expositions of the nature of science; above all, for the student to turn around often and look back on the way he has traveled, trying to understand what it is all about instead of merely knowing facts or rules soon to be forgotten. Some teachers claim that the history and nature of science *are* discussed in orthodox formal courses. But it seems certain that orthodox courses do not give sufficient time for this, and, by their very crowding and attention to preparation for later courses, give an erroneous picture of science. Thus, a general characteristic of the new science courses seems likely to be a great reduction of subject matter.

This reduction of subject matter has no intrinsic virtue. It is merely the key that opens the door to teaching schemes which do have special virtues.

Terminology. The Block-and Gap Scheme

To make discussion easier, I offer the following descriptions and labels. (See Fig. 1.) Let us represent the whole field of knowledge by a table of columns, as in Fig. 1(a), each column representing the subject-matter for a single science. For example, the column BCC'B' represents physics. The orthodox course proceeds straight down a column covering the subject matter as thoroughly as time and the students' preparation will permit, usually trying to lay a foundation

for later courses. Some colleges have several orthodox physics courses for students with different preparation or interests. These are sketched in Fig. 1(b) as alpha, beta, and gamma. Alpha is a "thin" or easy course which begins at the beginning and does a thorough job of mentioning topics but avoids the hard parts of the treatment. In physics, such courses are often recommended for students who have not studied physics before, and they are sought by many premedical students. They seem hard enough to their clients, but to their instructors they often seem too easy and too dull. In tests, easy numerical problems are more common than derivations involving argument, and the student's real understanding is not inquired into.

Beta is a standard "freshman course." All the important topics are treated in turn, often with little time to show their consequences or their interrelations. History is mentioned but not discussed and certainly not brought to life. The course is well packed with content. In physics courses of this kind, problem-solving still involves many arithmetical substitutions but contains some algebraic argument.

Gamma is a harder or more advanced course given in some colleges, for students with special preparation or interests. Note that since such courses trade on students' previous preparation, they can omit considerable patches of material, while expanding others and even treating them as case histories. (Curiously enough, a gamma course is often a specially hard version of the block-and-gap course suggested for general education. There is a useful moral in this for the future development of all courses.)

One type of course used for general education has been a "survey" course which at least mentions a large number of topics. Some claim that this gives valuable wide acquaintance; others condemn it as giving a smattering of facts with no time for either thorough treatment or discussion of ideas. We may sketch this in Fig. 1(c) by selecting some small patches all over the wide table of the sciences. If we like, we may draw a connecting thread linking the many topics in a pattern, but little blood is likely to flow along so thin an artery. I call this the "smörgasbord" course. (The title "survey course" has several meanings and is best avoided.) As thus described—a wild rush through a host of little topics—this is not the course any of us wish to claim as our own, not even the proponents of several kinds of "survey." We each disclaim it, and perhaps think it is the course the other fellow gives! We feel the smörgasbord course does too little to show the structure and methods of science. Another of its dangers is that it tends to glorify the "wonders of science," paint the romance in glowing colors, and make the scientist seem a wizard who dispenses knowledge and "explanations." Science courses should debunk such false claims.

A scheme better able to meet some of the requirements discussed runs more like the one marked "block-and-gap" shown in Figs. 1(d) and 1(e). This may be in a single science or in a mixture. Its essential characteristic is that large quantities of material are omitted outright, so that only some of the topics of an orthodox course such as beta are dealt with—say, half the topics. Those included are treated thoroughly as to subject matter (the blocks representing such topics are dense) and as to their background (the blocks extend out toward other topics and other sciences and are clearly related to other blocks). Connecting these blocks are lines along which flows the "lifeblood" of the course: discussions and investigations, historical studies, ideas and information carried from one block to another, and thence, enriched, to still another or back to the first one—showing the organic structure of science. The gaps are essential; they give room for the lifeblood to show interrelationships, and they reduce the content of the course so that there is time for discussion, time for ideas to sink in, and time for the student to look back and reconsider. Whatever our differences of aim and course structure, this name "block-and-gap" with its crude picture seems useful as a description of the kind of course we are trying to provide for general education.

Though they indicate something about the course structure, these pictures say little about the type of approach or treatment of the material which is selected, nor does the block-and-gap picture say what material is selected. In block-and-gap, the approach may be historical (as it may be in beta), perhaps through case-history studies or through readings of original texts; or

it may be empirical by experiment; or the course may be composed of selections from an orthodox course, with a change of attitude. Still we are likely to agree on some general characteristics of the block-and-gap course: restriction of topics, attention to their interrelations, and prominence to student discussion and creative and critical work.

Critics, seeing the sketches or the name "block-and-gap" for the first time, may complain that a set of unrelated topics lightly treated would make a poor course, a sort of emaciated survey. This is not what I intend the term to mean. The blocks are meant to be related, by pointing the moral from one to the other by referring back to earlier ones or prophesying forward to future ones, and their topics are meant to be treated seriously. In fact, the blocks should be both dense and extensive. The discussions should be thorough, explaining essential points thoroughly, and they should extend to neighboring material, pursuing some of it to an advanced stage. For example: an orthodox beta course in physics, will certainly treat the kinetic theory of gases, probably stopping at a prediction of Boyle's law and a brief mention of diffusion. In a block-and-gap course, kinetic theory may be selected as a block or it may be omitted; but, *if it is selected*, it is likely to be treated more slowly and thoroughly so that students have time to discuss its assumptions and to understand its mathematics (instead of learning the latter parrot-fashion for examinations), and to see its use as a theory. It is also likely to be treated more fully. If it is done at all, it is done well. Just saying, "There is a prediction of Boyle's law . . . Avogadro . . . diffusion . . . ," then hurrying on to a new topic, may teach something about kinetic theory but it is likely to leave the student little the better in his understanding of science in general. He needs to have time to discuss the parts played by assumptions and the mathematical machine. And he needs to see more of the results that the theory can yield.[3] If kinetic theory is studied in a block-and-gap course, it may well be continued through comments on molecular speeds, velocity of sound, and a discussion of gas viscosity—with an experimental test of the theory's surprising prediction that gas friction should be independent of the pressure. Here is theory being used and tested, then used to predict the breakdown of its own prediction.

Making a Block-and-Gap Course

In offering a block-and-gap course as part of general education, we try to show, by means of picked specimens carefully treated, what science is like, how scientific work is done, and what scientists are like as thinkers and workers. In preparing such a course the first thing we have to do is to make it clear to all who teach in it that the gaps are big, that they are not expected to cover all the ground of a course like beta. We should go further and assure them that there is no golden rule for choosing the blocks. "This is *your* course," I would like to say to my colleague. "Make your own choice of topics and do not try to copy mine. Be careful not to choose too many.[4] Make a selection which will have interconnections, showing some kind of a framework of science. Then you and your class can go ahead and think and argue and discuss and learn about science. At the same time, the class will learn some scientific material. Though you choose only a few topics, and though much of your attention and theirs is concentrated on methods and ideas, they will learn some facts and learn them well enough to seem well educated in science in the old-fashioned sense. And they will be happy to continue reading and learning for the rest of their lives."

Choice of Topics ("Blocks")

If we have reached some agreement about aims and decided on a block-and-gap course, we still have choices both of content and of approach or manner of treatment. These choices affect each other.

Many of us feel that the choice of topics or

[3] This is one of the more serious crimes of the orthodox courses, building up some theoretical treatment or general principles, and then not using them. Thermodynamics, in almost any general physics text, gives an excellent example of this crime.

[4] I would like to make a rule, saying "You are welcome to add topics of your own to the selection I am using, but for every one topic you add, you must delete at least one of my topics." When a teacher understands this rule, he often says, "In that case I do not mind so much about adding topics—I see it is the treatment of them that matters."

content is a matter of secondary importance. These are the blocks, chosen from one science or several, to form a scheme of subject matter with some interconnections. In this choice of content, there are no *indispensable* topics, but when we are making our own choice it is helpful to study the choices made by others. So I hope that all who construct block-and-gap courses will publish lists of topics with some details of treatment and comments on their success.[5] In studying such lists we should avoid making our lists too big by trying to include everyone else's suggestions! An examination of some block-and-gap courses in action shows that their material content is about 50 percent of that of an orthodox course.

Most physicists are unrelenting in their choice of physics as the best field for a block-and-gap course in general education. It offers such good blocks, well-constructed on a foundation of experiment, well-documented in their history, well-connected to each other. Chemistry, at first sight a healthy rival, is likely to prove less suitable. Too many chemical facts must be acquired before theories can be formulated and used to make the facts make sense. Is is the second round of chemistry that is so good, when the beginner has already run right through the book gathering facts, and starts at the beginning again, ready to use atomic theory, equations, etc., as he goes. To meet this difficulty many an orthodox chemistry textbook begins with pictures of atomic structure, announcing atomic and molecular theories, with rote-rules for equation balancing. For general education we do not welcome this scheme of announcing things long before their experimental basis can even be seen—the "hand-out" system. It is common enough in physics courses (see any textbook) but it is more easily avoided in physics than in chemistry. The life-sciences present different teaching problems and different benefits. We should like to see their block-and-gap courses offered parallel to physical-science ones.

In physical-science courses the choice between several sciences and a single one is not so severe as it sounds. Comparing two actual single-science courses, one in chemistry and the other in physics, I find at least 30 percent of the topics are common to both. And the physics course extends into astronomy as well as chemistry till it is hardly distinguishable from a "physical-science," mixed-science block-and-gap course.

Treatment of Topics in Block-and-Gap Courses

The block-and-gap idea provides only a useful admonition concerning the making of a course. The choice of approach or treatment of topics is is more important and interesting. This choice depends partly on the weight given to various aims, partly on the teaching staff and equipment available. I am sure there is no single ideal or "right" method. There is much good in historical treatment—with the right teacher. President Conant's "case-history" method can be a delight to both students and teachers. For some students, orthodox presentation of material seems best. This enables students to cover a block rapidly leaving considerable time for discussion. Used alone this treatment might develop into beta, but mixed with other treatments it can be a healthy accelerator. The use of laboratory work raises new problems. An efficiently run laboratory may be deadly, or it may be the heart of the course. Even examinations can set the tone of the course.

How shall we choose the blocks? How shall we run the course? What examinations shall we have? How can teachers be trained for such courses? All these problems relate to the underlying flavor of the course, its real spirit, on which its success depends. Insisting on a block-and-gap format merely clears the ground for a healthy spirit to grow. Can we then write another general prescription to foster the growth of that spirit, a prescription that will not restrict us to any particular approach but will yet guide the course? I think we can. This, the *major* general problem of the new courses, will be discussed in a later paper.

[5] When I discuss lists with colleagues I find no single topic labeled "absolutely essential," not even the conservation of energy; but some aspects of scientific method—for example, the use of hypotheses—are common to all. However, some topics seem to be favorites. Several physical science block-and-gap courses include among their blocks: Kepler's laws, Joule's work, atomic theory.

A Classroom Model of Vertical Ionospheric Reflection*

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THE behavior of a radio wave in the ionized regions of the earth's atmosphere is controlled by the index of refraction of the medium. This index, neglecting (1) double refraction because of the earth's magnetic field, (2) the minor effects due to the presence of heavy ions, and (3) collisional friction, is

$$\mu = v_g/c = [1-(Ne^2)/(\pi m f^2)]^{\frac{1}{2}} \tag{1}$$

when

$$Ne^2/(\pi m f^2) \leqq 1 \tag{2}$$

and where μ is the index of refraction, v_g is the group velocity of the signal, c is the velocity of light, N is the density of free electrons, e is the electronic charge, m is the electronic mass, and f is the frequency of the radio wave.

In the simplest case of ionospheric reflection, at normal incidence on a homogeneous horizontal layer, a signal travels upward until it reaches a level where $\mu=0$ and then returns to its source. The time of transmission varies with the height at which μ becomes zero and with the distribution of ionization found below that level. Both of these factors are shown by Eq. (1) to be functions of the frequency of the signal.

This behavior is analogous to that of a ball thrown vertically upward, but the analogy can be made more apt by controlling the "downward" acceleration of the ball in such a way that the acceleration is a function of the "height." This can be done by rolling the ball on a curved and inclined surface.

Consider a ball sliding without friction across a horizontal surface against a hillside, as in Fig. 1. On the horizontal surface the velocity of the ball is v_0 and its total energy with respect to that surface is $\frac{1}{2}mv_0^2$. At a point on the hillside where the ball has climbed to a height h, the velocity is v and the energy is $\frac{1}{2}mv^2+mgh$. Therefore

$$\tfrac{1}{2}mv_0^2 = \tfrac{1}{2}mv^2 + mgh, \tag{3}$$

or

$$v^2 = v_0^2 - 2gh, \tag{4}$$

and

$$v/v_0 = [1-(2gh/v_0^2)]^{\frac{1}{2}}. \tag{5}$$

This relation has the same form as that of Eq. (1) so we may set, with any convenient scale factors

$$\left.\begin{array}{c} 2g \approx e^2/\pi m \\ h \approx N \\ v_0 \approx f \text{ and } c \end{array}\right\}. \tag{6}$$

We must now examine the velocity in a practical model. In this case the ball must roll, not slide, and it is convenient to provide a track to guide it. The simplest form of track is a milled slot, as shown in Fig. 2. A homogeneous ball of radius r rolling in such a slot with a translational velocity v and angular velocity ω has a kinetic energy

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2, \tag{7}$$

where $I=\frac{2}{5}mr^2$, $\omega = v/r_1$, and r_1 is the distance from the center of the ball to the surface along which it rolls.

If such a ball be rolled down a starting chute from an initial height h_0, its velocity v_0 on the horizontal plane is given by

$$mgh_0 = \tfrac{1}{2}mv_0^2 + \tfrac{1}{2}[\tfrac{2}{5}mv_0^2(r^2/r_1^2)] \tag{8}$$

or

$$v_0 = \left(\frac{2gh_0}{1+\frac{2}{5}d^2/(d^2-w^2)}\right)^{\frac{1}{2}}, \tag{9}$$

since

$$r^2/r_1^2 = d^2/(d^2-w^2), \tag{10}$$

w being the width of the slot in the guiding surface and d the diameter of the ball.

Correspondingly, the velocity v at a height h

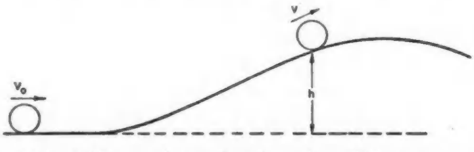

FIG. 1. Nomenclature for discussion of the energy of a ball on a hill.

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is found from the energy relation

$$\tfrac{1}{2}mv_0^2+\tfrac{1}{2}\left[\tfrac{2}{5}mv_0^2(r^2/r_1^2)\right]=\tfrac{1}{2}mv^2 + \tfrac{1}{2}\left[\tfrac{2}{5}mv^2(r^2/r_1^2)\right]+mgh, \quad (11)$$

and [as in Eqs. (3), (4), and (5)],

$$\mu=\frac{v}{v_0}=\left(1-\frac{2g}{1+\frac{2}{5}d^2/(d^2-w^2)}\cdot\frac{h}{v_0^2}\right)^{\frac{1}{2}}. \quad (12)$$

Thus the first relation in Eq. (6) is changed only in scale, in the case of the rolling ball, and the others are unaffected. In the present case

$$\frac{2g}{1+\frac{2}{5}d^2/(d^2-w^2)}$$

occupies the place of $2g$ in the former solution.

A model can therefore be made, subject to four primary defects mentioned below, in which the behavior of a rolling ball is similar to that of a packet of radio frequency energy in the ionosphere. The ball is simply rolled along an approximate graph of the density of ionization (plotted vertically) against height (plotted horizontally).

The four important defects in the model are:

1. The velocity v of the ball on the hillside is parallel to the slope, not parallel to the plane of v_0. This error may be minimized by the use of a "flat" model with small slopes.
2. The initial velocity represents both frequency and velocity of light. Therefore a varying time scale must be used when changing from one frequency to another.
3. The gyroscopic effect of the rolling ball affects the path in the case of a three-dimensional model. This effect does not appear in the case of vertically incident reflection considered here.
4. If the ball is to be continuously guided in the vertical plane by the contoured surface, the curvature of the surface must not require downward accelerations greater than that due to gravity. This limit is severe and, like defect No. 1 above, leads to the use of a flat model. If, as is most convenient, the initial velocity v_0 is achieved by rolling the ball down an incline from a height h_0, the limit can be easily established. At any point on the height-density curve the magnitude of the centrifugal acceleration is v^2/R, where R is the radius of the curvature of the path. The velocity v at that point is $(2gh_1)^{\frac{1}{2}}$ where h_1 is the difference between the starting height h_0 and the height h at the point in question. The transverse acceleration is therefore $2gh_1/R$ and the ball will fail to follow the track if $2h_1$ is greater than R.

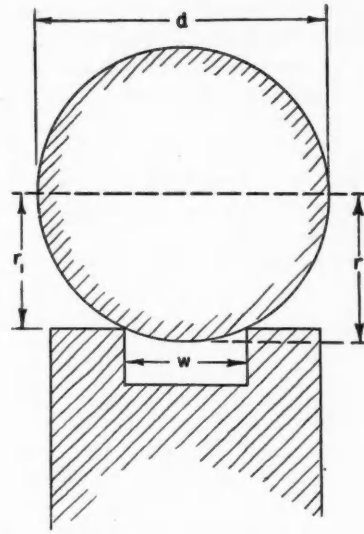

FIG. 2. Nomenclature for use in treating the rotational and translational energy of a ball on a track.

A practical model has been made to accommodate a ball bearing two inches in diameter and to display the phenomena of normally incident ionospheric reflection on scales of distance and time suitable for classroom demonstration. This model is shown in Fig. 3. It was made from a 12 foot length of 1 in.×1 in. steel[1] with a slot $\frac{5}{16}$ in. wide by $\frac{1}{8}$ in. deep milled in the center of the upper surface. The bar was bent cold, after milling, to the shape shown (with a vertical exaggeration of 10) in Fig. 4. After bending, the top surface of the bar was filed smooth to remove surface pitting and the edges of the slot were slightly rounded with fine emery paper. It was originally intended to temper the model as an aid in the reduction of friction, but the process would be difficult and might well lead to serious distortion. Since the friction is satisfactorily

[1] Chromium-molybdenum steel, SAE 4140.

small (the ball rolling the whole length of the bar and returning within $\frac{1}{2}$ in. of the starting point) the only difficulty that would be avoided by heat treatment is a tendency to dent the edges of the slot by careless handling of the ball. This trouble has not been serious but it is anticipated that occasional filing down of the surface may be desirable.

The model is mounted on three pedestals of adjustable height, one at the center and one near each end. Since the bar is somewhat flexible, this permits recovery of the original calibration when the model is set up on any reasonably horizontal surface. The method found most satisfactory is to adjust the starting end until the ball will just rest (or move very slowly without acceleration) at the zero point representing the surface of the earth, and to adjust the other end until the ball can barely be made to rest in the very slight hollow beyond the maximum of the F_1 layer. This hollow has been made vanishingly small to avoid the danger of trapping the ball as can be done in the region between the E and F_1 layers. If the ball be rolled with an initial energy that just permits it to cross the E maximum in the "upward" direction, it will lose enough energy through friction in passing from the E maximum to the F_1 layer and back to prevent its returning to the starting point. This is an embarrassing occurrence in a demonstration as there is no ionospheric analogy for it. It is easily avoided by treating the E layer group retardation in a slightly cursory way, avoiding the region (about $\frac{1}{8}$ in. wide) on the starting chute that corresponds to the frequency range in which trapping is possible. A detailed exposition of the retardation and of the transition from one layer to another can then be given in the region of the F_1 maximum where the danger of trapping is negligible. In fact it is possible to find a setting of the adjustable starting guide (shown in Fig. 3) such that chance will determine whether the ball reflects from the F_1 layer or passes through it to the F_2 layer.

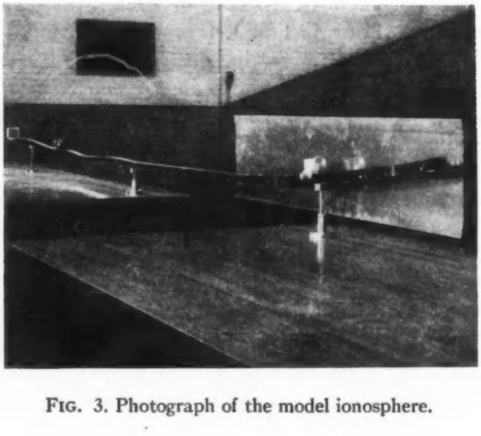

FIG. 3. Photograph of the model ionosphere.

An easily removable shield has been provided to conceal the starting chute from the class, if desired. In the absence of this shield, confusion may arise because only a part of the structure is actually the model. Its use emphasizes the fact that the method for obtaining the initial velocity is not critical. The edge of the shield also serves admirably to mark the point corresponding to the surface of the earth. A bumper lined with rubber foam is attached to the end of the model opposite the starting chute. This serves to reverse the translational and rotational energy of the ball and return it to the operator after penetration of the entire ionosphere. The concept of penetration is made clear by painting the model in contrasting colors on opposite sides of the F_2 maximum.

To obtain easy control of the initial velocity v_0, it is wise to devote about a third of the total length of the bar to the starting chute. A degree of elegance is provided by making the frequency scale on the chute linear with the initial displacement of the ball. This is done by letting the coordinates of points along the chute satisfy the following relations

$$y=\alpha f^2$$

$$s=\beta f$$

$$x=\frac{1}{4\alpha}\left\{2[(\alpha y)(\beta^2-4\alpha y)]^{\frac{1}{2}}+\beta^2\sin^{-1}\frac{2(\alpha y)^{\frac{1}{2}}}{\beta}\right\}, \quad (13)$$

where s is the distance along the chute from the zero point, y is the vertical distance above ground level, and x is the horizontal distance from the zero point.

In the model shown in Figs. 3 and 4 the constants are:

$$\alpha=0.0303 \text{ inch/(megacycle)}^2$$
$$\beta=3.937 \text{ inches/megacycle.}$$

For the model ionosphere itself the constants are approximately:

Vertical scale: 2.40 inches
$=10^6$(free electrons)/cm^3
Horizontal scale: 1 inch
$=2$ miles above the surface of the earth.

The coordinates of points on both the starting chute and model proper are given in Table I.

Although the model is intended only for demonstration purposes, a reasonably accurate graph of virtual height of reflection *versus* frequency can be drawn from it experimentally, thus suggesting that it might form the basis for an undergraduate laboratory experiment. The technique is simply to roll the ball from a series of starting points and measure the time required for the ball to return to the surface of the earth. Since the standard deviation of the time is of the order of 0.1 second and the interval itself averages about ten seconds, the measurements can be made easily by stop watch.

A graph of the results of such an experiment is shown in Fig. 5 where the indicated experimental points are each the average of five measured intervals. Because it is easier, as suggested by the figure, to measure the initial displacement of the ball along the curve of the starting chute rather than vertically, an accurate calibration of the height *versus* displacement was made at the time the model was constructed. The values, of course, differ from those of Table I only by accidental variations due to the difficulty of bending the bar with precision.

The measured values are transformed by the scale factors stated above through the following relations:

From Eq. (9), the velocity of light is:

$$v_0=23.4(h_0)^{\frac{1}{2}} \text{ inches/second in the model}$$
$$=75.3(h_0)^{\frac{1}{2}} \text{km/second in the full scale} \quad (14)$$

where h_0 is measured in inches.

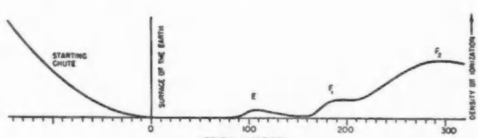

FIG. 4. Profile of the model ionosphere with a vertical exaggeration of 10 to 1.

TABLE I. Coordinates (in inches) of the classroom model ionosphere (all measured from the point identified as the surface of the earth).

Frequency in Mc	Starting chute x	Starting chute y	Ionosphere model* x	Ionosphere model* y	Ionosphere model* x	Ionosphere model* y
0.0	0.00	0.00	0 to 26	0.00	64	0.71
0.5	−1.97	.01	28	.04	66	.78
1.0	−3.94	.03	30	.15	68	.88
1.5	−5.91	.07	32	.27	70	1.03
2.0	−7.87	.12	34	.31	72	1.20
2.5	−9.84	.19	36	.29	74	1.38
3.0	−11.81	.27	38	.24	76	1.56
3.5	−13.77	.37	40	.19	78	1.74
4.0	−15.73	.48	42	.14	80	1.90
4.5	−17.70	.61	44	.10	82	2.03
5.0	−19.67	.76	46	.08	84	2.16
5.5	−21.63	.92	48	.06	86	2.25
6.0	−23.59	1.09	50	.06	88	2.32
6.5	−25.55	1.28	52	.17	90	2.37
7.0	−27.51	1.48	54	.42	92	2.40
7.5	−29.46	1.70	56	.63	94	2.40
8.0	−31.42	1.94	58	.70	96	2.38
8.5	−33.37	2.19	60	.70	98	2.34
9.0	−35.32	2.46	62	.69	100	2.27
9.5	−37.27	2.73				
10.0	−39.21	3.03				
10.5	−41.16	3.34				
11.0	−43.10	3.67				

* These values are intended to represent a "standard" daytime ionosphere obtained by adding the Chapman distributions for three layers having the following characteristics:

Layer	Maximum density of ionization	Height of maximum ionization	Scale height
E	1.3×10^5 free electrons/cm^3	110 km	10 km
F_1	2.5×10^5	185	10
F_2	1.0×10^6	300	50

The frequency is obtained from the scale factor for the density of ionization through the relation [derived from Eq. (1) by setting $\mu=0$]

$$N=1.24\times10^{-8}f^2, \quad (15)$$

which yields

$$f_{mc}=5.75(h_0)^{\frac{1}{2}}.$$

The virtual height is defined as the height to which a signal would have gone had it traveled always at the velocity of light and been reflected by a perfect mirror. It is, of course, the quantity measured in ionospheric work at vertical incidence. In the case of the model, it is:

$$h'=37.6t(h_0)^{\frac{1}{2}}$$
$$=6.55tf, \quad (16)$$

where h' is in kilometers, h_0 is in inches, t is in seconds, and f is in megacycles.

The result of this transformation of the data of Fig. 5 is given as the solid curve of Fig. 6.

The cusps marking the transition from one layer to another are better represented than in most actual records. The transition at about 5 megacycles is, as mentioned above, peculiarly satisfying as an infinitesimal change in frequency produces a change of about two to one in the virtual height.

The dotted curve of Fig. 6 is given for purposes of comparison and is taken from an earlier publication.[2] It was decided to reduce the E layer critical frequency for this model from about four to 3.5 megacycles, but in other respects the curves should coincide. The deviations in the general virtual height level and in the critical frequency of the F_1 layer are evidences of the limited accuracy with which the model was made.

This somewhat pedantic discussion has not, it is hoped, concealed the fact that this model is a fascinating toy and an excellent tool for instruction. In the writer's experience it seems capable of conveying the simpler facts of ionospheric behavior and methods of research in a fraction of the time required at a blackboard.

The ordinary technique in a demonstration is, after introducing the analogy, to roll the ball repeatedly at gradually increasing velocities. As the frequency approaches the critical value for the E layer, the existence of group retardation becomes obvious, and this delay clearly accounts for about a third of the total transmission time when the ball nearly comes to rest at the E layer maximum. From this point a slight increase in frequency (enough, however, to avoid the danger of trapping that was discussed above) allows the reflection to occur from the *side* of the F_1 layer, but with a very large contribution to the transmission time occurring in the slow passage through the E layer. Another increase, this time of 10 to 20 percent, in the frequency then reduces this group retardation to the point where it becomes inconspicuous without involving any great change in the actual height of

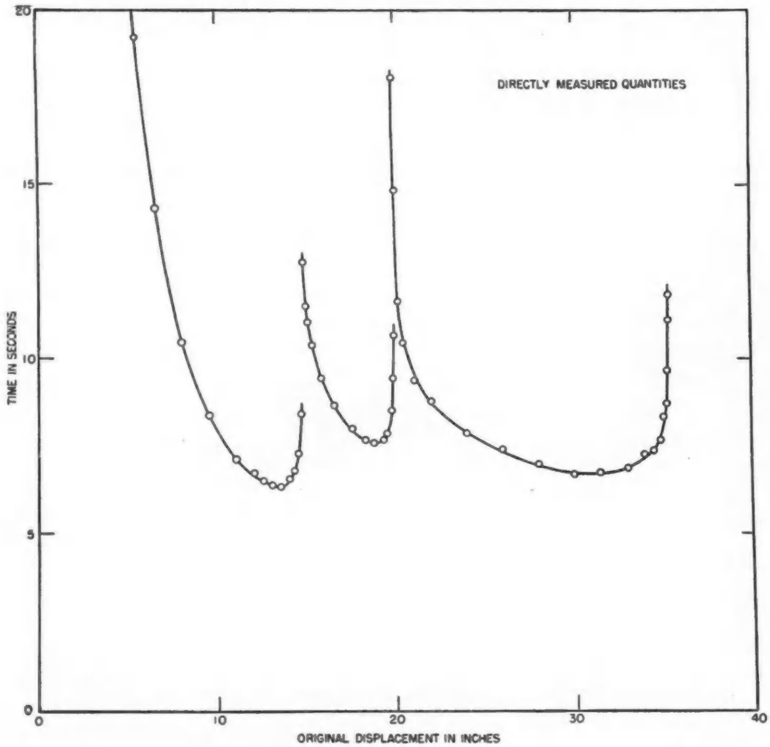

FIG. 5. Model data relating the time of transmission to the point at which the ball starts rolling. The abscissa is the initial displacement of the ball from the foot of the starting chute, and the ordinate is the time taken by the ball to ascend the model and return to the foot of the chute.

[2] J. A. Pierce, "The true height of an ionospheric layer," *Physical Rev.* **71**, 698 (1947).

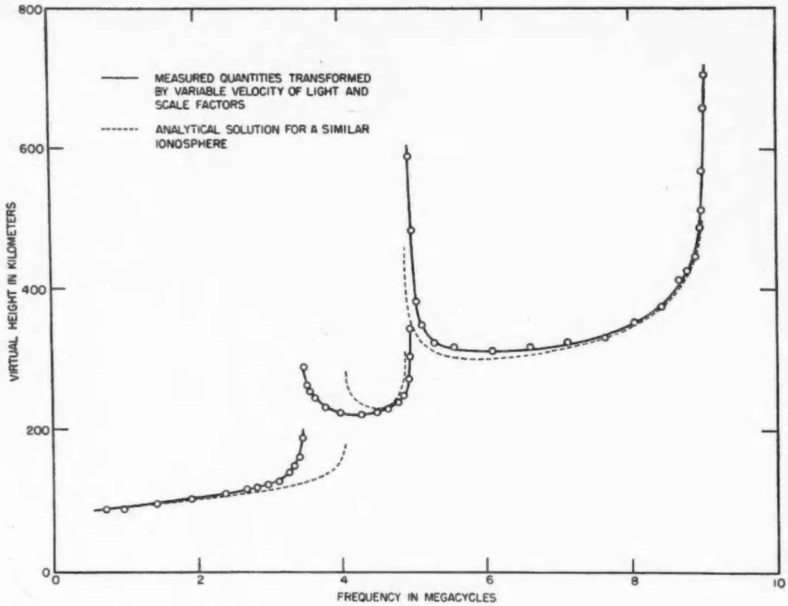

FIG. 6. The data of Fig. 5 transcribed into the virtual height/frequency pattern. The abscissa is proportional to the square root of the initial vertical elevation of the ball on the starting chute and the ordinate is proportional to the measured time of Fig. 5 multiplied by the abscissa of this figure. The constants of proportionality are derived in the text. The experimental curve should agree with the analytical solution except in the region from 3 to nearly 5 megacycles.

reflection. Still further increase in frequency produces noticeable F_1 layer retardation, until the F_1 critical frequency is reached.

At this point a very delicate adjustment of frequency produces the most elegant phenomenon of all, the passage from the F_1 maximum to the *side* of the F_2 layer (a distance of some six inches, corresponding to 20 km or more), at a velocity which may be made as small as 5 percent of the velocity of light. Again, further increases of frequency immediately reduce the group retardation to negligibility until the ball is climbing well to the top of the F_2 maximum. Here the final conspicuous slowing of the group velocity is seen and presently an increase to more than the F_2 critical frequency causes penetration of the entire ionosphere. In this region the reduction of the velocity in the F_2 layer is conspicuous unless the frequency is raised distinctly above the critical value. Thereafter the ball traverses the entire model at substantially the velocity of light. These observations may be supported by slides of the form of Figs. 5 and 6.

After an introduction of this sort it is easy to call upon the students' imagination to explain the phenomena of oblique incidence transmission since it is obviously only the velocity component toward the hill which would cause the ball to climb over a three-dimensional structure of this sort. The variation of limiting frequency with angle of incidence and even the Pedersen ray behavior (when the ball would nearly climb over the hill and then run for some distance almost parallel to and just below the crest before falling back towards the earth) can be explained qualitatively before being developed analytically. It appears probable to the writer that the facts learned in this way will be retained easily and may provide a valuable steadying influence when the more detailed behavior of a radio wave in an ionized medium is presented in mathematical terminology.

Adjacent-Axes Charts from Ordinary Graph Papers

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RECENTLY Burrows[1] described a number of methods for using graph papers to facilitate various types of numerical calculations. Although his treatment was rather comprehensive, there are other methods which may well be added to those which he listed. It is the purpose of this paper to show how ordinary graph papers may be used in the construction of the convenient adjacent-axes charts for representing two variables connected by a unique relationship. There are a number of general types of two-variable equations for which adjacent-axes charts are easily made from available graph papers.

The simplest and most familiar of these is the type where the two variables are linearly related when plotted on linear (uniform-scale) graph paper

$$y = mx + b, \tag{1}$$

where m and b are constants. This equation gives a straight line with a y-intercept equal to b and with a geometrical slope equal to m, provided the same unit of distance is used for the scales on both the y and x axes. If the scales are so chosen or constructed that the unit of distance along the x axis is m times as long as the unit of distance along the y axis, the resulting straight line has a geometrical slope of one and a geometrical inclination of 45 degrees. Then these x and y axes may be put side-by-side to form an adjacent-axes chart where the position of one scale with respect to the other is fixed by the requirement that when $x = 0$, $y = b$.

A typical example of Eq. (1) is the familiar Fahrenheit-centigrade temperature conversion relation

$$F = 9/5C + 32. \tag{2}$$

When plotted with the same unit of length used to represent both the centigrade and Fahrenheit degree, in Fig. 1(A), the geometrical slope is 9/5. In Fig. 1(B) the centigrade scale has been expanded until its unit of length is 9/5 that of the Fahrenheit unit and the geometrical slope is one. These centigrade and Fahrenheit scales may now be placed side-by-side, noting that when $C = 0$, F must $= 32$, to form the adjacent-axes chart of Fig. 1(C).

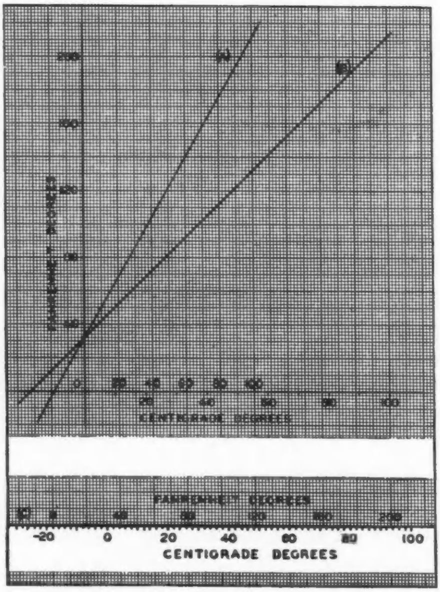

FIG. 1. Representation of $y = mx + b$, (A) with geometrical slope of 9/5, (B) with geometrical slope of 1, and (C) in adjacent-axes form.

A more complicated and more interesting type of two-variable relationship conveniently representable by adjacent-axes charts is the family of equations of the form

$$y = ax^p, \tag{3}$$

where a and p are constants (positive or negative, fractions or integers). Taking logarithms of both sides reduces Eq. (3) to a more convenient form

$$\log y = p \log x + \log a. \tag{4}$$

Here $\log y$ is a linear function of $\log x$, and Eq. (4) is seen to be of the form of Eq. (1), with $\log x$ replacing x and $\log y$ replacing y. It is apparent

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[1] W. H. Burrows, *Am. J. Physics* **17**, 114–126 (1949).

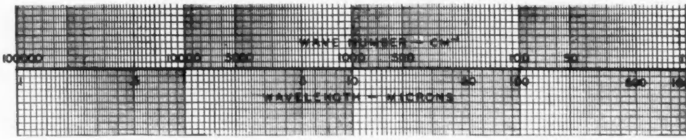

FIG. 2. Wavelength to wave number conversion: $\nu\lambda = 10{,}000$.

that if Eq. (3) were plotted on ordinary full logarithmic paper, the result would be a straight line with an intercept on the $x=1$ axis equal to a and with a geometrical slope equal to p, provided the same unit of distance (length of log cycle) is used for both y and x. As before, this straight line may be given a geometrical slope of one by an expansion or contraction of one of the scales until the length of the x-cycle is p times the length of the y-cycle; these y and x axes may now be put side by side to form an adjacent-axes chart. The number of x-cycles required in the chart is determined by the range of x. The number of y-cycles is p times the number of x-cycles, while each x-cycle is p times as long as a y-cycle. If the sign of p is negative, the scales for y and x must be reversed with respect to each other. The relative position of the scales is determined by the a of Eq. (3) such that $y=a$ when $x=1$ (i.e., when $\log x = 0$).

Construction of adjacent-axes charts to represent relations of the form of Eq. (3) can be illustrated by a few charts which the author has had occasion to use recently. The first is a chart to give a rough conversion of wavelengths in microns to wave numbers in cm^{-1} over the range from long ultraviolet to short microwaves. The equation involved, put in the form of Eq. (3), is

$$\nu = 10{,}000\lambda^{-1}. \tag{5}$$

Even this simple relation is clearer when written in the log form

$$\log\nu = -\log\lambda + \log 10{,}000. \tag{6}$$

The resulting adjacent-axes chart is shown in Fig. 2. Note that the range of λ, from about 0.4 to about 1000 microns, indicates that for the λ-scale a strip of log scale four cycles long is needed; the value of the coefficient of $\log\lambda$ (unity) indicates that the λ- and ν-cycles should be the same in number and length; the sign of the coefficient of $\log\lambda$ (negative) indicates that the λ- and ν-scales should be reversed; and the position of the λ-scale with respect to the ν-scale is fixed by the constant term, such that $\nu = 10{,}000$ when $\lambda = 1$. Equation (5) was especially easy to handle because the coefficient of λ was not only an integer but also had the particular value, one.

Other special cases of Eq. (3) which are also easy to handle include those for which p is any ratio involving the numbers of cycles in which semilog (or full log) graph papers are available, as 1, 2, 3, 4, 5, and 7. Consider the equation:

$$d = 18.2F^{\frac{1}{2}}, \tag{7}$$

where d is a diameter in microns and F is a flow rate in arbitrary units (Fig. 3). This was the calibration relation found to express the diameter of copper spheres, over the range from 10 to 200 microns, in terms of the flow rate through a particular Stokes' law air column particle separator. Because its range is somewhat greater than one cycle, a strip of two-cycle semilog paper is chosen for the d-scale. From the exponent of F it is seen that the number of d-cycles required is 1/2 the number of F-cycles needed; therefore a strip of four-cycle paper is chosen for the F-scale. Since the exponent of F is positive, the log cycles of the two scales run in the same direction. The relative position of the scales is fixed by the constant 18.2, such that the point, $d = 18.2$, is placed opposite the point, $F = 1$.

As a still more involved case consider the adiabatic pressure-volume relationship

$$pv^{\gamma} = k. \tag{8}$$

In the form of Eq. (3) and for a particular gas and set of experimental conditions this might be written

$$p = 20v^{-1.4}, \tag{9}$$

where p is the pressure in atmospheres and v is

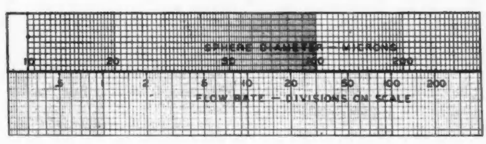

FIG. 3. Adjacent-axes representation of $d = 18.2F^{\frac{1}{2}}$.

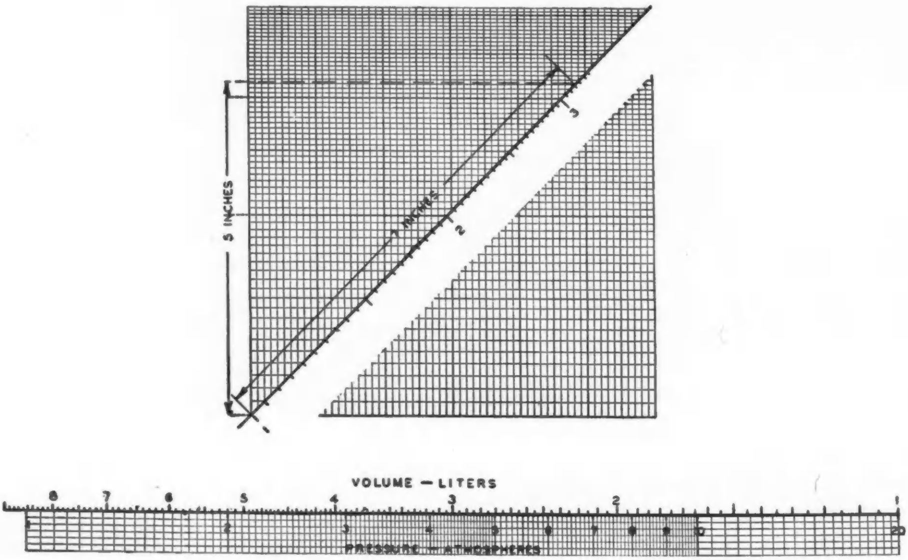

FIG. 4. Construction of expanded scale from given log scale. Adjacent-axes representation of $pv^{1.4}=20$.

the volume in liters. Suppose it is desired to have an adjacent-axes chart to express the volume in terms of the pressure over the range from 1 to 20 atmospheres. Immediately, the range of p indicates that a strip of slightly more than one cycle of log scale is needed for the p-scale. Since the number of p-cycles is 1.4 times the number of v-cycles, it is apparent that a single v-cycle will be adequate. Also the length of a v-cycle must be 1.4 times the length of a p-cycle; this v-scale is produced from the p-scale by simple construction of a log scale expanded by the factor 1.4. Such a construction is seen in Fig. 4 where a 5-in. section of log scale is expanded into a 7-in. section. Since the exponent of v is negative, the p- and v-scales are reversed with respect to each other. Their relative position is fixed by the constant 20, such that $p=20$ when $v=1$.

Other types of two-variable equations may be represented by adjacent-axes charts made from combinations of ordinary graph papers. As an example consider the equation:

$$y=ae^{px}, \tag{10}$$

where a and p are constants. Upon taking logs of both sides it is apparent that charts over any desired range of y and x can be made from suitably chosen strips of semilog and linear graph papers.

It should be emphasized that the important advantage of the adjacent-axes chart over the usual rectangular-axes graph lies in the fact that it avoids the inconvenience and possibility of error associated with following co-ordinate lines to and from a curve.

In summary, the basic principle underlying adjacent-axes charts is simply this: choose scales which, if used as ordinary rectangular co-ordinate scales, would result in a straight line with a geometrical inclination of 45 degrees. With this understanding and a few sheets of each of the several ordinary graph papers (linear, semilog or full log, probability, etc.) it is easy to construct adjacent-axes charts to represent any two-variable relationship from among a rather wide range of types.

Conversion Charts

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THESE charts are designed to help a beginner use physical units in several systems. Conversion factors often cause confusion in the minds of beginners, for they do not know whether to multiply or divide by the factor. By using a chart, the direction of increase of the quantity is made somewhat clearer. Conversion factors have not been included here because very complete tables have been published elsewhere.[1–4]

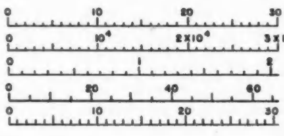

Fig. 1. Mass, *m*. In the absolute system (the mks system is an absolute system) mass is a primary quantity. In the gravitational system, the mass of a body is equal to the force acting on the body divided by the acceleration which is produced by that force. If this force is the weight of the body, the mass equals the weight divided by the acceleration due to gravity. ($m=W/g$.)

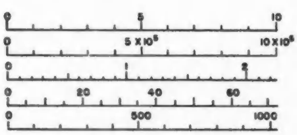

Fig. 2. Force, **F**. In an absolute system, the force acting on a body is the product of the mass of the body times the acceleration produced by that force. ($\mathbf{F}=m\mathbf{a}$.) In a gravitational system, force is a primary quantity and is given by the force (or weight) which a standard body exerts in the field of gravity.

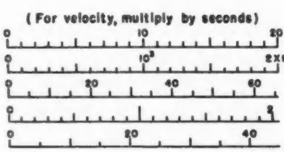

Fig. 3. Acceleration, **a**, and velocity, **v**. Acceleration is the ratio of change of velocity to the corresponding interval of time. ($\mathbf{a}=d\mathbf{v}/dt$.) Velocity is the ratio of the change in position to the corresponding time interval. ($\mathbf{v}=d\mathbf{s}/dt$.)

[1] Smythe, *Static and dynamic electricity* (McGraw-Hill, New York, 1939), pp. 533–536.

[2] Moon and Spencer, "Utilizing the mks system," *Am. J. Physics* **16**, 25 (1948).

[3] Harnwell, *Principles of electricity and electromagnetism* (McGraw-Hill, New York, 1938), p. 605.

[4] *Handbook of chemistry and physics* (Chemical Rubber Publishing Co., Cleveland, 1948, ed. 30), pp. 2369–2412.

On all the charts, quantities on the same vertical line are equivalent. The left side of each chart is taken as the smallest value of the quantity (usually zero), and the chart runs far enough from zero to indicate one or more possible practical cases where the quantity is used. Values of certain well-known constants, such as the permeability of free space and the heat of vaporization of water, are shown on the appropriate charts. If the charts do not give a usable range for some fields, such as atomic physics, all of the values in a given chart can be multiplied by any power of 10, and the chart is still accurate. For instance, to convert the charge of an electron (1.60×10^{-19} coulomb) the chart for electric charge should be multiplied by 10^{-19}.

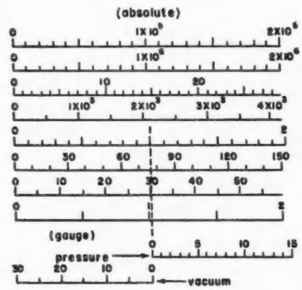

Fig. 4. Pressure, *p*. Pressure is force per unit area. ($p=F/A$.)

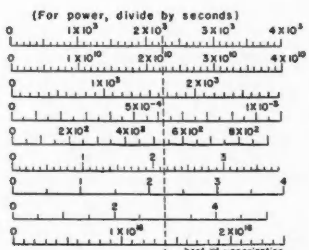

Fig. 5. Work or Energy, *U*, and Power, *P*. The work done by a force is the product of the force times the distance which it has moved. ($U=\int\mathbf{F}\cdot d\mathbf{s}$.) Power is the energy used in a time interval divided by the time interval. ($P=dU/dt$.) In addition to the units derived from a self-consistent system (joules, ergs, and ft pounds), many other units of energy and power are commonly used and are shown.

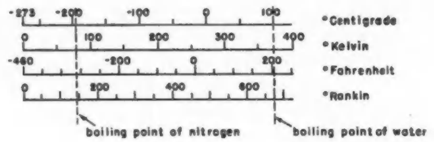

FIG. 6. Temperature, T. The unit of temperature on the centigrade and Kelvin scales is 1/100 of the change in temperature between the melting point and the boiling point of water and 1/180 of this difference on the Fahrenheit and Rankine scales. The melting point of water is designated by 0° centigrade and 32° Fahrenheit. The Kelvin and Rankine scales start with 0 at the absolute zero. Interpolation between these values is defined by means of the heat exchange in a reversible heat engine operating between the given temperature and the melting point of water. However an international scale employing resistance thermometers, thermocouples, and blackbody emission has been established, and it conforms with the thermodynamic scale as closely as is possible with present knowledge. (See, for instance, Cork, *Heat* (Wiley, New York, 1942, ed. 2), pp. 35–38.)

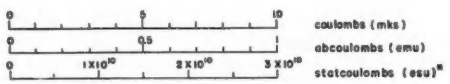

FIG. 7. Charge, Q. In the mks system as commonly used, charge is a primary quantity along with mass, length, and time. In esu, charge is the product of the square root of the force exerted by two equal charges upon each other in a vacuum times the distance between the two charges. ($Q=F^{\frac{1}{2}}r$ or $F=Q^2/r^2$.) The asterisk indicates Gaussian unit.

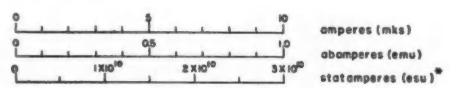

FIG. 8. Current, I. The current flowing through a surface is the charge transported across the surface divided by the corresponding time interval. ($I=dQ/dt$.) The asterisk indicates Gaussian unit.

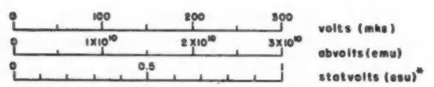

FIG. 9. Potential Difference, V. The potential difference between two points is the amount of work required to move a charge from the first to the second point divided by the charge moved. ($V=U/Q$.) The asterisk indicates Gaussian unit.

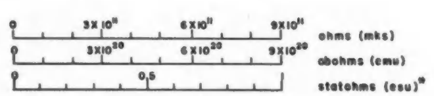

FIG. 10. Resistance, R. The resistance of an element is the ratio of the potential difference across the element to the current flowing through it. ($R=V/I$.) The asterisk indicates Gaussian unit.

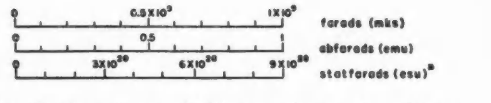

FIG. 11. Capacitance, C. The capacitance of an element is the ratio of the charge on it to the potential to which the element is raised by the charge. ($C=Q/V$.) The asterisk indicates Gaussian unit.

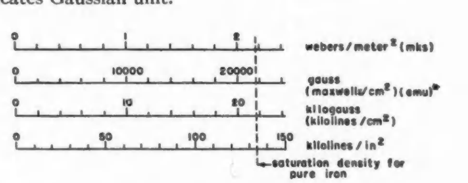

FIG. 12. Magnetic flux density, **B** (or magnetic induction). The magnetic flux density at a point is proportional to the force on an element of current placed at the point and has a direction perpendicular to the current element and to the force. ($d\mathbf{F}=Id\mathbf{s}\times\mathbf{B}$.) It is the quantity which is determined by an experiment in which the magnetic fields are altered and the induced voltage in a coil is measured. The asterisk indicates Gaussian unit.

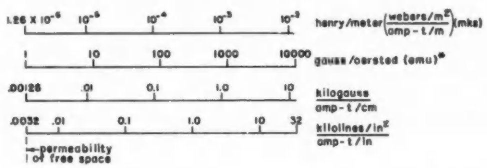

FIG. 13. Permeability, μ. The permeability of a medium is the product of the relative permeability of the medium times a dimensional factor, the permeability of free space. The relative permeability of a substance is the ratio of the magnetic flux density when all space is occupied by the substance to its value at the same point in a vacuum, the configuration of all circuits and the magnitude of all currents being the same. (See reference 1, p. 251.) The permeability of free space is defined as unity in the cgs emu system and is experimentally found to be $4\pi\times10^{-7}$ henry/meter in the mks system. A good discussion of the electric and magnetic quantities in the mks system is given in Houston, *Principles of mathematical physics* (McGraw-Hill, New York, 1948, ed. 2). The asterisk indicates Gaussian unit.

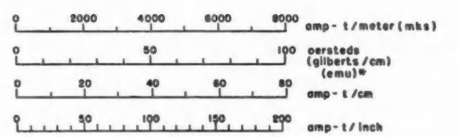

FIG. 14. Magnetizing force, **H** (or magnetic field intensity). The magnetizing force is a quantity which is calculated from the known currents, permeabilities and geometry of the system. It is equal to the flux density divided by the permeability of the medium. The asterisk indicates Gaussian unit.

The mks system of units occurs on the first line in all charts, and the cgs Gaussian system occurs on the second line. Various other com-

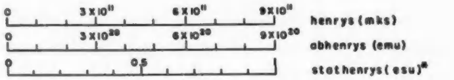

FIG. 15. Inductance, L. The inductance of an element is the ratio of the flux linking the element to the current flowing in the element. ($L = N\Phi/I$.) The asterisk indicates Gaussian unit.

monly used units follow. The Gaussian units are marked with an asterisk (*) on the charts of electrical and magnetic quantities. Whenever a unit is called by several different names, the most common name is given first, and other names are included in parentheses.

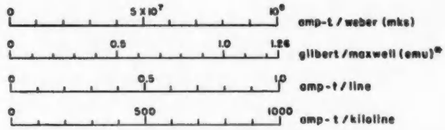

FIG. 16. Reluctance $\mathcal{R}$. Reluctance finds its principal use in engineering applications. It is the ratio of the magnetomotive force to the flux in a magnetic circuit. The magnetomotive force ($\mathfrak{F}$) between two points is the line integral of the magnetizing force, i.e., $\mathfrak{F} = \int \mathbf{H} \cdot d\mathbf{s}$, $\mathcal{R} = \mathfrak{F}/\Phi$. The asterisk indicates Gaussian unit.

The authors wish to acknowledge their gratitude to J. E. Goldman for helpful criticisms of the manuscript.

The Philosophies of Science of Eddington and Milne*

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THE most revolutionary work in science since the work of Newton has been published in recent years by the late Sir Arthur Eddington and by Professor E. A. Milne. Their results are so astonishing that scientists in general have been content to remain skeptical, when not actually hostile. This is due in part to the highly mathematical character of their presentation and also, in part, to the new philosophies of scientific method which they advocate. These new philosophies of science (they differ in important respects) require a total readjustment of outlook on the part of scientists who have been trained in the ordinary way and who have so far managed to avoid the problems of philosophy, and even pretend that they are no concern of the scientist.

It must be admitted, however, that these new philosophies are not as clearly presented or as convincing as one would wish. This is partly because their authors are treading new ground and are, so to speak, building their philosophies as they proceed. Attempts to clarify various points often lead to apparent defects. Nevertheless, I do not think that any scientist can afford to neglect these recent advances.

It is a well-known dictum of philosophy that no one philosopher ever understands another. There is, no doubt, a great deal of truth in this remark, and, as I do not profess to understand clearly either Eddington's or Milne's philosophy, while at the same time being convinced of the outstanding importance of their work, there is only one course open to me. I shall try to describe the results which they have achieved, and then put forward considerations which seem to me such as to make their significance clear.

Let us start by being clear about the subject under discussion. Eddington's and Milne's theories are chiefly about *metrical science*, that is to say, about that part of science which is concerned with measurement. Thus they are, for instance, very relevant to astronomy, which is a science that depends almost wholly on measurement, but much less relevant to zoology. And let us be clear also, at the outset, that pure *counting* is not measuring in the sense that this word is used here. Measuring always involves counting but it also involves the observation of coincidences of certain marks.[1]

In order to understand the kinematic relativity of Milne we must first recall the history of the evolution of geometry. This began, as the derivation of the word geometry implies, with measurement of material things. The Greeks turned

* The substance of a lecture given at the Consejo Superior de Investigaciónes Científicas, Madrid, May 1948.

[1] See also the author's "A new treatment of electric and magnetic induction," *Proc. Physical Soc.* **52**, 577 (1940) and "A new treatment of dimensions," *Proc. Physical Soc.* **53**, 418 (1941).

geometry into an abstract deductive system, but proofs were still aided by the consideration of material things, such as diagrams and solid bodies, and it was for long looked upon as providing reliable knowledge of the physical world, being a rational elaboration of measuring operations. Doubt does not appear to have arisen until the seventeenth century, and followed the work of Descartes, who discovered how to transform geometrical forms into algebraic equations, which amounted to converting the axioms and theorems of geometry into purely arithmetical propositions. When it was later shown that the axiom about parallel lines could be replaced by others which led to different but equally self-consistent systems, it became clear that geometry, like syllogistic logic, was a purely formal study, and the question as to whether its definitions, axioms, and conclusions were true of the physical world was a matter for experimental determination.

The theory of relativity may be described as a "geometrization" of physics, if the word "geometry" is extended to include time, and although it is not a satisfactory theory, we can look upon it as suggesting the possibility that *physical* laws may be deduced from a set of definitions and axioms, in a similar manner to the proofs of geometry, and that the question of their applicability to Nature is again a matter for experiment. Steps in this direction had already occurred in physics, when a large number of physical laws were shown to follow from the assumption of certain simple principles, such as the principle of the conservation of energy, and the principle of least action, but these were taken to be all-embracing properties of Nature rather than simplified axioms in a deductive system.

Now Milne, in his "Kinematic Relativity," has attempted to derive the laws of physics, by starting with abstract concepts and by postulating certain axioms of an epistemological nature. He starts with hypothetical observers who can experience the irreversible passage of time, and who are provided with clocks which they can adjust into "congruence" by means of information obtained by exchanging signals. Conventional definitions of distance are made from the time interval of reflection of a signal sent and received back by an observer. This replaces the physical assumption of a rigid measuring rod. Other principles are invoked, which are epistemological rather than physical (such as that all observers are equivalent), but Milne has not yet succeeded in eliminating all appeals to quantitative experience. He has to accept the three spatial dimensions as a fact.

Nevertheless, Milne makes less appeal to experience in his axioms and definitions than any other investigator, and succeeds in deriving laws of motion, gravitation, electromagnetism, and some others. This astonishing achievement—even though it is obviously imperfect—makes it very much more difficult for the ordinary physicist to dismiss the new and revolutionary suggestion that metrical physics can be presented as a rigidly deductive system, as independent of support from the laboratory as geometry is from the drawing board. Moreover, any criticism that Milne's observers, signaling their clock readings to one another over large distances, are not found in nature, is as irrelevant to "Kinematical Relativity" as the criticism that points and straight lines are not found in nature, is irrelevant to geometry. The only need for contact with experience is when we try to *identify* physical quantities with the mathematical expressions which occur.

It has long been known, of course, that Euclid's geometry could only be exemplified to the limits of accuracy of physical measurement on the drawing board. As soon as we apply it to large material objects it fails, since these objects bend under their own weight. Beams of light supplied the deficiency, until it was shown that they bend near a large body such as the sun. Now we realize that Euclid's geometry is not a rationalization of measurement so much as an *idealization*, and care is needed in its application. It can only be used with confidence in regions where it has previously been shown *by experiment* to apply.

Up to the present, Professor Milne has not succeeded in throwing any light on the origin of quantum phenomena, and there is nothing in his theory with which to identify it. In this respect "Kinematic Relativity" falls short of the all-embracing character of Sir Arthur Eddington's philosophy of metrical science, which we must consider next.

Eddington's position can be most clearly stated

in the following way.[2] He realizes that the *metrical* knowledge of the universe consists of numbers produced by pointer-readings, and asserts that a physicist requires only a scale, a clock, and one eye. Now we can only correlate such information by means of a mathematical *structure*. The experience of mechanical pressure, an electric shock, a taste, a smell, and so on, cannot be introduced into a system of pointer-readings. Granting, then, that all that theories of metrical science can produce is structure, we are faced with the intriguing possibility that perhaps we already know enough to discover the underlying structure, and if so, once we have discovered it, we ought to be able to predict all the possible metrical relationships to be found. Eddington has developed what he calls a wave-tensor calculus, a form of sedenion algebra, which he claims is the desired representation of the structure of the metrical aspects of the external world, as embodied in our present theories.

Eddington, like Milne, makes use of certain epistemological principles, but he is more willing than Milne to make use of nonquantitative results of physical theory, such as the equivalence of mass and energy, the exclusion principle, etc., and this enables him to achieve more. His fundamental epistemological principle is that *every item of physical knowledge is an assertion of what has been, or would be, the result of carrying out a specified experimental procedure.*

He maintains, further, that all his results are derived from the condition that "the conceptual interpretation which we place on the results of measurement must be consistent with our conceptual interpretations of the process of measurement. Having examined critically our conceptual interpretation of the process of measurement we have to define symbols with properties that correspond precisely to the conceptions introduced."[3]

Epistemological considerations which affect our construction of theories are, for instance, the method of analysis, and the view that analysis should lead ultimately to identical permanent units, so that all apparent variety is due to different structure. This manner of thinking is, of course, the basis of the atomic theory.

Since Eddington does not restrict himself to purely epistemological considerations, his first step in his pursuit of world-structure must be to unify the theories of the macroscopic and microscopic universe, that is, his relativity theory and the theory of quantum mechanics. The first attempt of this sort had been made by Dirac in 1928 with the linear equation of the electron, his object being to discover a form of equation which would be invariant for rotations and Lorentz transformations while at the same time satisfying quantum conditions. Tensor calculus was looked upon as the ideal tool for treating problems of invariance and covariance, but Dirac, adopting methods of his own, produced an expression of very unsymmetrical appearance which he showed to be invariant for transformations of special relativity. This type of expression had been missed in ordinary tensor calculus owing to an arbitrary convention, which Eddington had noted at the time.[4] In applying tensor theory to physical measurements, its basic vector was identified with a geometrical displacement $(dx)^\mu$. By changing this so that the basic vector was identified with Dirac's four-valued quantity ψ, Eddington obtained a new calculus which he called *wave-tensor calculus*, in which the invariance of the wave equation is included. He also adopts the quantum-mechanical device of representing physical quantities by operators.

Another mathematical device which aided Eddington in his solution of the problem of finding a symbolic device for representing the metrical structure of the World, was the calculus of quaternions which Sir William Hamilton invented in 1843. This calculus is another instance of the reduction of geometry to algebra, but in this case it is a noncommutative algebra with four principal units, and certain geometrical relations can be very simply described by its use. A vector (a, b, c) for instance is represented by

$$\mathbf{i}a+\mathbf{j}b+\mathbf{k}c$$

where a, b, and c are ordinary numbers (which could be obtained by measurement) and $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ indicate that the measurement is along the x,

[2] The reader is reminded that the author is responsible for presenting both Eddington's and Milne's theories in what *he* considers the most convincing way.

[3] Eddington, *Fundamental theory* (Cambridge University Press, 1946), p. 266.

[4] Eddington, *Mathematical theory of relativity* (Cambridge Univ. Press, 1924), p. 49.

the y, and the z axes, respectively. The symbols $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ and their relations ($\mathbf{ij}=\mathbf{k}$, $\mathbf{jk}=\mathbf{i}$, $\mathbf{ki}=\mathbf{j}$, $\mathbf{ijk}=-1$) represent the *structure*—in this case three-dimensional space. Now the various relativity theories, as we have noted, amount to a geometrization of macroscopic physics, and so Eddington had to try to discover a calculus which would symbolize the structure and properties of a multidimensional generalized physics in a similar manner to that in which quaternions represent metrical relations in three dimensions.

Eddington starts building his *world-structure* from the following consideration: a measurement involves four entities; that is to say, it is a number obtained from the association of four entities. In measuring a length we must observe the coincidence of *two pairs* of marks, and in measuring a time a *single pair* of marks must be viewed at *two* different instants. All physical quantities can be measured by length and time measurements.[5] Numbers associated with four entities, Eddington calls *measurables*.

The simplest entities have neither parts nor magnitude and correspond to points in Euclid's geometry. Their only property is existence or nonexistence in the structure contemplated. Eddington associates with them a symbol whose meaning only becomes definite when conjoined with another symbol representing the structure, and which has only two eigenvalues, representing existence or nonexistence.

Thus the most primitive measurable is provided for in this scheme by four entities whose existence attributes are independent, and consequently a measurable has $2\times2\times2\times2=16$ eigenvalues. This array is called a *tensor measure*. Measurables with the same tensor measure have the same quantitative properties but they are made qualitatively different by means of associated symbols, just as the different spatial directions are distinguished in quaternion algebra. This is in accordance with the view that variety is due to the structure and not to the elements. It is achieved mathematically by a "rotation group:" Eddington chooses from the known rotation groups the one in which the rotations are symbols with 16 eigenvalues. This group defines what he calls the "$E-F$ frame" and the tensor measures are now taken to be "$E-F$ numbers."

Our metrical knowledge regarding an electron, say, is represented now by an expression of this type

$$\sum_\mu E_\mu p_\mu$$

where the p_μ's are ordinary numbers, and the E_μ's represent the physical quantity. The need for F symbols as well, is due to the fact that some quantities are defined with reference to other similar quantities—for instance, the unit of electric charge. The elementary charge e is not something which you could ever rub off your sleeve with a fountain pen; it is, as Eddington has profoundly remarked, a measure of interaction, and in particular, for simplicity, a measure of interaction between two equal charges. The E symbols represent one, the F symbols the other.

From the sedenion algebra so developed, Eddington suggests, the whole of metrical physics can be deduced—not only the equations but also the constants. In order to do this, Eddington, like Milne, has, of course, to make use of a process of *identification:* he has to find a tensor possessing mathematical properties resembling those of a given physical quantity. For example, a tensor identified with energy must have the property of conservation, and the structure characteristic to be looked for in indentifying "truly elementary particles" (such as electrons), is *idempotency*, i.e., if the E symbol for an electron is S then $S^2=S$. There seems to be a certain amount of latitude in this identification, and at one time, at least, Eddington spoke of "lumber"—those tensors which appeared to have no physical counterpart in the world of observation. Dimensionless constants such as m_p/m_e, the ratio of the mass of the proton to that of the electron, and $hc/2\pi e^2$, the fine-structure constant, are clearly independent of any arbitrary *unit*, but they do depend "on our conceptual interpretation of the process of measurement," and consequently, if the symbolic structure (the sedenion algebra) is consistent with this interpretation, these numbers ought to be able to be wrung from it by sufficiently acute insight. This is what Sir Arthur Eddington has achieved in a most remarkable way.

But these constants are constants of the quantum-mechanical theory only; there are other

[5] See the author's "A New Treatment of Dimensions," Ref. 1, a treatment with which Sir Arthur Eddington was in agreement.

constants belonging to cosmological theory; and as Eddington claims to have united the two theories with his wave-tensor calculus, he ought to be able to establish relation between these constants. For this purpose it is necessary to find a problem that can be treated rigorously by both methods. Such a problem is the equilibrium state of a nonradiating self-contained system of a very large number of particles. In macroscopic theory this was first attempted by Einstein, with his spherical universe of radius R depending on the total number of particles N. To keep this "universe" together, he introduced *ad hoc* a constant —the cosmological constant λ. When it was found that the universe is expanding, Einstein proposed to drop it; but to Eddington this was "an incredibly retrograde proposal," and he uses λ as a measure of mutual repulsion. In microscopic theory, a radiationless steady system is said to be in its "ground state." Making use of the exclusion principle, Eddington assumes that in the ground state the N particles occupy the N states of lowest energy. The total energy of the "universe" is thus found in terms of R and N and the quantum constants, and this should equal the energy calculated from macroscopic theory making use of the relation—energy equals mass multiplied by the velocity of light squared.

In this way Eddington unites the gravitational constant G, and λ, with m_p, m_e, and e (the charge on the electron). He compares this achievement with Maxwell's unification of electromagnetism and optics, by showing that the ratio of the emu to the esu was always a power of c, the velocity of light—"the elimination of superfluous constants is an outward sign of the unification achieved . . ."[6]

Before comparing Eddington's calculated values in Table I with those obtained by observation, it should be noted that, as some of them depend on the arbitrary centimeter, gram, and second, he has to introduce these values somewhere. This he does by measured values of c, of the Rydberg constant for hydrogen, and of the Faraday constant for hydrogen.

Eddington's value for N—"the total number of particles in a spherical world composed of hydrogen and which satisfies requirements of quantum theory"—is obtained in more than one way, and with close agreement. The number has the same status in physics as Avogadro's number—that is to say, no one has counted it, and no one is expected to. It is a number obtained from measurements and calculated according to a certain theory. If the theory is rejected, then the number loses its meaning. In his last book, which is not complete owing to his death, Eddington mentions the contributions which his work makes to nuclear theory. For example, it leads to the determination of the law and constants of the non-Coulombian energy of two protons, the mass-defect of deuterium and helium, the separation constant of isobaric doublets (and hence of the radii of nuclei), and other important relations.[7] Masriera has suggested recently that the total number of elements, on Eddington's theory, is 96.[8]

These very astonishing achievements of Eddington and Milne cannot be dismissed, and yet it is very difficult to present their philosophies of science in a clear and acceptable form. Both make use of *identification* in order to establish a connection between elements of their deductive systems and the physical quantities known to the ordinary physicist. Eddington originally started frankly with symbols—with *relata* and *relations*—

TABLE I. Calculated and observed values of the atomic constants.

Symbol	Description	Calculated	Observed
m_e	Mass of electron	9.10924×10^{-28} g	9.1066
m_p	Mass of proton	1.67277×10^{-24} g	1.67248
e	Elementary charge	4.80333×10^{-10} esu	4.8025
h	Planck constant	6.62504×10^{-27} erg sec	6.6242
m_p/m_e	Mass ratio	1836.34	1836.27
$hc/2\pi e^2$	Fine-structure constant	137	137.009
G	Constant of gravitation	$6.6665\ \times10^{-8}$ $cm^3/g\ sec^2$	$6.670\ \pm0.005$
V_0	Nebula recession	572.36 km sec^{-1} mega-$parsec^{-1}$	560
$\mathfrak{M}$	Magnetic moment of H′ atom	2.7899 (nuclear magnetons)	2.7896 ± 0.0008
$\mathfrak{M}_N$	Magnetic moment of neutron	1.9371 (nuclear magnetons)	$1.935\ \pm0.02$

[6] Eddington, *Relativity theory of protons and electrons* (Cambridge Univ. Press, 1936), p. 4.

[7] Ref. 3, p. 211.

[8] *Memorias de la Real Academia de Ciencias and Artes de Barcelona* 28, 143 (1946). *Anales de la Real Sociedad Española de Física & Química* B, 7, 795 (1948).

their structure has "some significance in regard to the ultimate structure of the world . . . it does not matter much what significance,"[9] but in his last book, he introduces *measurables* associated with four "entities." Milne claims that the measurements he describes his fundamental observers as making, could in principle be carried out, but as his identifications are made with galaxies, the time to carry out a to-and-fro signal would be greater than ordinary observers could spare!

It is not clear that anything is gained by attempting to give some physical justification for the initial steps in raising a deductive structure. It is not necessary in quaternion algebra to give a physical explanation of the symbols: all that is necessary is to give clear definitions and axioms; then we later *identify* certain forms with geometrical relations and show that all geometrical relations can be simply derived from it. Similarly, it would seem that we now have to admit that the internal consistency of our system of dynamics, electrodynamics, and quantum theory, is the consequence of our interpretation of the results of measurement, and not of the measurements themselves. Its internal consistency is no more dependent on measurement made with galvanometers, spectroscopes, and other instruments, than is the internal consistency of geometry on the results of measurement with scales. This consistency is *mathematical*, and mathematics, according to the late Professor A. N. Whitehead, is "the science concerned with the logical deduction of consequences from the general premises of all reasoning."[10]

Eddington appears to accept an idealist philosophy as an explanation, with the "mind" forcing its habits and prejudices onto the external world: Milne and his followers quote Shakespeare and the Bible. Would it not be preferable to attribute their results to the measuring conventions which have been accepted uncritically as means of introducing numbers into Nature? It is to a searching examination of the implications of these conventions, together with general presuppositions (such as that all variety is due to structure), that I suggest we should look for an explanation of the revolutionary new outlook which we shall have to adopt toward the results of metrical physics.

[9] Ref. 4, p. 213.

[10] Article on Mathematics, *Encyclopedia Britannica*, 11th ed.

New Members of the Association

The following persons have been made *members* or *junior members* (*J*) of the American Association of Physics Teachers since the publication of the preceding list [*Am. J. Physics* **17**, 525 (1949)].

Allen, C. Crosby, Apt. 2C, 227 Sullivan St., New York 12, N. Y.
Allen, Roland, 965 New Scotland Rd., R. D. Slingerlands, N. Y.
Ashley, Ernest Newton, Jr., Dept. of Physics, West Virginia State College, Institute, W. Va.
Christensen, George Maurice (*J*), c/o Clayton and Lambert Mfg. Co., 1701 Dixie Highway, Louisville, Ky.
Conant, George Herbert, Jr., Box 516, Idaho Springs, Colo.
Dillon, John A., Jr., 72 Knowles Street, Pawtucket, R. I.
Loomis, Robert Simpson (*J*), 426 Welch Avenue, Ames, Iowa.
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Luehrmann, Robert Oscar, 2318 8th St., Great Bend, Kan.
McQuitty, Jim Bob, 118 Anderson, Columbia, Missouri
Marcaccio, William, 312 View St., Covington, Ky.
Martin, Willment Plunkett, N. A. I. Northrop Field, Hawthorne, Calif.
Rainwater, L. James, Physics Dept., Columbia University, New York 27, N. Y.
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Rhodes, Joseph Elmer, Jr., School of Physics, Georgia Institute of Technology, Atlanta, Ga.
Scheel, Brother Nivard, C. F. X. (*J*), Xaverian College, Silver Spring, Md.
Schwetman, Herbert D., Physics Dept., Baylor University, Waco, Tex.

Microwave Experiments and Their Optical Analogs

GORDON FERRIE HULL, Jr.
Dartmouth College, Hanover, New Hampshire

THE advent of vacuum tubes for the production of microwaves and fixed crystals for detecting them have made it possible to demonstrate readily all the optical properties of electromagnetic radiation. The apparatus which will be described in this paper makes use of radiation of wavelength 3.2 cm and the experiments are essentially those for free space microwaves, demonstrating the analogous phenomena of geometrical and physical optics. Because of the fact that a microwave generator produces plane polarized, coherent, monochromatic radiation, one would expect some differences to occur between the optical properties of free space microwaves and those of light. This is indeed the case, for a light source is seldom strictly monochromatic; its wavelength is less by a factor of 10^5 and its radiation is not coherent. Consequently, the optical analogs of microwaves must be considered as analogs and not as identities.

Generator and Receiver for 3.2-cm Microwaves

The generator and receiver for 3.2-cm (1.26-in.) microwaves are shown in Fig. 1. A Western Electric 2 K 25 or 723 A/B reflex klystron is used as the microwave source.[1] This tube requires two power supplies, one regulated at 300 v and about 30 ma, to accelerate the electron stream through the cavity, and another to apply a variable negative voltage from 0 to -300 v between the cavity and repeller. A means for modulating the repeller voltage with an audio oscillator should also be provided. This tube will deliver about 30 mw power, in a frequency range from 8500 to 9700 Mc (3.5 to 3.1 cm). The coaxial output of the 723 A/B tube is coupled to standard $\frac{1}{2}\times 1$ in. (outside dimensions) rectangular wave guide having 0.050-in. wall thickness. The TE_{01} mode[2] is excited in the rectangular guide and the microwaves are propagated along it and radiated from the 20-db horn which is shown in Fig. 1 mounted together with the 723 A/B tube and its section of wave guide on a wooden bench. Wave guide mounts for the 723 A/B tube and also power supplies can be bought from suppliers of microwave apparatus or can be made. The details of construction of the wave guide mount is given in the Western Electric circular describing the operation and performance of the 723 A/B tube.

The receiver which is shown clamped to a rod stand in Fig. 1 consists of a short section of wave guide into which a Western Electric 1N23B fixed crystal detector is appropriately mounted. This piece of wave guide is shorted at one end and has a plane rectangular coupling flange at its open end. The crystal output is connected with coaxial microphone cable to either an audio amplifier and loudspeaker for demonstration or to a vacuum tube voltmeter if precision measurements are required. If the crystal is "square law" which is usually the case, the vacuum tube voltmeter will measure directly the relative microwave power received. Although such a receiver can be built, it is simpler to buy a wave guide crystal mount from one of the suppliers of microwave apparatus. Since the receiver is small, it can be used as a probe for exploring radiation coming from different directions. It can be held in the hand for this purpose and moved about. Since the TE_{01} mode has the electric vector across the short dimension of the wave guide, the receiver is also an analyzer for the polarization of the microwave radiation. Finally the receiver can be mounted on a wooden bench similar to the one on which the transmitter is mounted. Other sections of wave guide and horns can be attached to the receiver by means of the coupling flange.

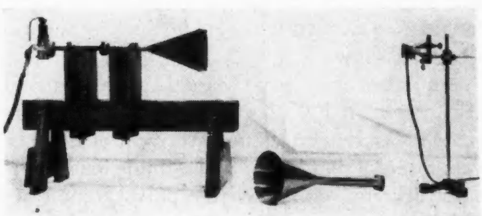

FIG. 1. Transmitter and receiver with horn radiators for 3-cm microwave experiments.

[1] J. R. Pierce, *Bell Lab. Rec.* **23**, 287 (1945).
[2] J. C. Slater, *Microwave transmission* (McGraw-Hill, 1942), Chap. 3.

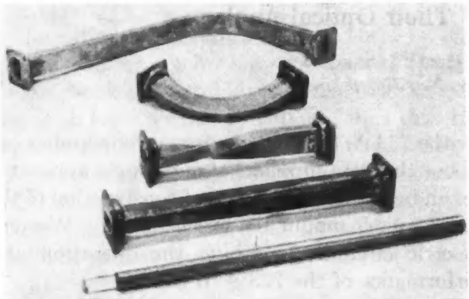

FIG. 2. Wave guide components for demonstrating transmission of 3-cm microwaves through wave guides.

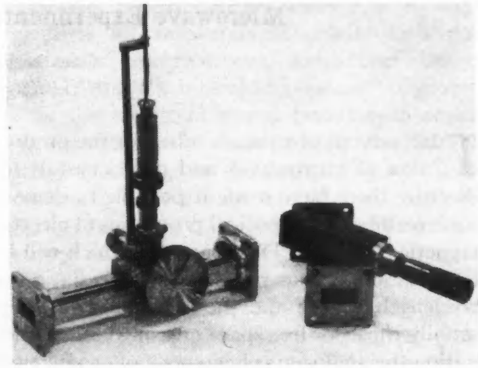

FIG. 3. Standing wave detector and wave meter for 3-cm microwaves.

In Fig. 1 two horn radiators are shown: a rectangular one attached to the generator on the wooden bench and a circular one in the foreground.[3] Each horn has an absolute gain of 20 db. The rectangular horn, which is attached to standard $\frac{1}{2}\times 1$-in. rectangular guide, has the dimension of $3.6\lambda \times 4.45\lambda$ for its open end and an axial length of 6λ, where λ is the free-space wavelength. The circular horn has a diameter of 4.4λ for its open end and an axial length of 6λ, and is attached to standard 1-in. outside diameter circular wave guide having a 0.032-in. wall thickness. Each of these horns has a total beam width of around 8° at the half-power points. The circular horn can be substituted for the rectangular horn on the transmitter, and the TE_{11} mode[4] will be excited in the circular wave guide. If this substitution is done, a standing wave will be produced in the section of rectangular guide from the generator because of the sharp discontinuity at the rectangular-circular wave guide junction. For demonstration purposes the discontinuity is not troublesome. However, for many measurements, it is desirable to eliminate the standing wave. This can be done by the insertion of a transition section of guide which changes gradually from rectangular to circular wave guide. Such a transition section is shown in Fig. 2.

A number of other wave guide components are also shown in Fig. 2 which are useful in wave guide measurements. The lowermost of these is a $\frac{1}{2}$-in. internal diameter circular guide loaded with a polystyrene rod. When the polystyrene rod is removed, the wave guide diameter is below cut-off and hence the microwaves are not transmitted through it. Next is the transition section from rectangular to circular guide, followed by a twist section to change the polarization through a right angle, and last, two rectangular wave guide bends. In Fig. 3 are shown a 3-cm standing wave detector and a wave meter. With the apparatus shown in Figs. 1–3, the experiments with wave guides previously described by the author for 10- and 20-cm microwaves can be performed with 3-cm microwaves.[5]

Transmission and Reflection

Besides the transmission of 3-cm microwaves through the various wave guide components shown in Fig. 2, the transmission of free-space microwaves through various dielectrics such as sheets of glass and plywood can be demonstrated by inserting them between the transmitter and receiver. These dielectrics are also partial reflectors. For these demonstrations the receiver should be at some distance from the horn radiator. A sheet of copper or $\frac{1}{8}$-in. mesh copper screening are excellent reflectors and prevent the transmission of the 3-cm microwaves. Standing waves in air can be produced by reflection from a copper sheet and an approximate wavelength measurement made. A plywood sheet which has been wound with wire spaced $\frac{1}{8}$ in. apart completely stops and reflects 3-cm microwaves when the wires are parallel to the electric vector, but

[3] G. C. Southworth and A. P. King, *Proc. I.R.E.* **27**, 95 (1939).
[4] Ref. 2, p. 162.
[5] G. F. Hull, Jr., *Am. J. Physics* **13**, 384 (1945).

transmits the radiation when the wires are at right angles. If the wire is spaced ½-in. on the plywood sheet, it will be found to be about half-reflecting and half-transmitting when the wires are oriented parallel to the electric vector.

Another interesting demonstration is to fasten a piece of "Z_0 cloth" in a wooden frame of such thickness that, if a brass sheet is placed against the frame, it will be exactly $\lambda/4$ from the Z_0 cloth. This combination is a perfect absorber of microwaves. The Z_0 cloth can be made of canvas painted with Aquadag so that it will have a resistance of 377 ohms per square, or can be bought. When this cloth is backed by a perfectly conducting sheet placed $\lambda/4$ from the cloth, the combination matches the impedance of free space, namely 377 ohms in the mks-unit system, and absorbs all the microwave energy. Later we shall see that a number of analogs exist between electrical transmission line theory and optics when we make use of the impedance presented to electromagnetic waves by free space, and by dielectrics both natural and artificial.

Instead of using a sheet of brass as a reflector, one can also use an ordinary plane mirror of silvered glass. It is easy to show for microwaves as for light that the angles of incidence and reflection are equal. Two silvered mirrors or two brass sheets at right angles will reflect the microwave radiation in the direction from whence it came. Finally a concave spherical mirror, if large in aperture, will focus the microwave radiation sharply. The author uses a silvered-glass, concave mirror 12 in. in diameter and 20 in. in focal length for this purpose. For short focal lengths and large diameters, parabolic reflectors of metal are used.

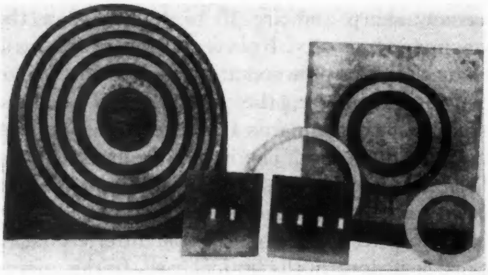

FIG. 4. Double and quadruple slits and zone plates for demonstrating interference and diffraction of 3-cm microwaves.

Interference and Diffraction

Because of the long wavelength of microwaves compared to light, interference from double- or multiple-slits can be demonstrated with large slits spaced only a few wavelengths apart. In Fig. 4 two brass plates 10 in. square are shown with two and four slits, respectively. The slits are $\lambda/2\times\lambda$ in size and are 2λ apart. By making the slits narrow, the diffraction pattern due to a single slit covers a total angle of more than 180° and has little effect on the interference pattern produced by the slits. To show the interference from two slits, the brass plate is placed in a holder clamped to the wooden bench supporting the transmitter and a few inches from the open end of the horn. The receiver can then be moved about in front of the double slit screen to locate the maxima. Besides the central maximum, there are maxima at 30° and 90° on either side of the central maximum because the slit spacing is 2λ. Another brass plate with two slits spaced 4λ apart will give four maxima on either side of the central maximum. Finally the brass plate shown with four slits spaced 2λ apart gives an interference pattern which is the combination of the two double slit patterns in which the slit spacings are 2λ and 4λ.

As a corollary to Young's double-slit experiment, interference can be produced by means of reflection as in Lloyd's single-mirror experiment. All that is required is to place a brass plate just to one side and in front of the horn radiator and then investigate the interference pattern with the receiver.

In Fig. 4 two zone plates are shown for demonstration of Fresnel diffraction. The fixed zone plate on the left has even-numbered half-period

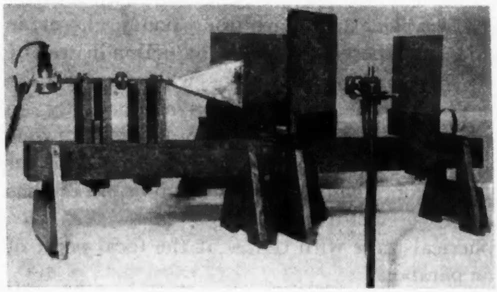

FIG. 5. Michelson interferometer for 3-cm microwaves.

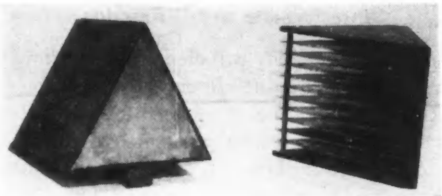

FIG. 6. Paraffin and wave guide prisms for 3-cm microwaves.

zones cut from sheet galvanized iron and tacked to $\frac{1}{2}$-in. thick plywood board, thus exposing odd-numbered, half-period zones. The zone plate on the right has four zones cut from sheet galvanized iron which are supported on a plywood board with two pins at the top of each zone. These zones are removable so that the effect of removing successive zones one after the other can be demonstrated when the receiver is placed at the focal point of the zone plate. Each zone plate has a focal length of 10λ (12.5 in.) and the radii of the zones are given by the usual equation $r_n=[nf\lambda+(n\lambda/2)^2]^{\frac{1}{2}}$, where n is the number of the zone, f the focal length, and λ the free-space wavelength. The zone plates should be placed at some distance from the transmitter to insure a plane wave striking the zone plate. When this is done the focal point is very sharp and can be easily located within $\pm\lambda/2$.

One might wonder why a zone plate is not generally used for the microwave antenna. The reason is that half the microwave energy is lost by reflection from the covered zones, assuming that the area of each zone is the same as that of any other zone and the gain of an antenna is 4π times the area of the antenna in square wavelengths.[6] Furthermore the phase difference of the energy emerging from successive exposed zones is on the average one wavelength, but not exactly one wavelength for arbitrary points chosen in adjacent exposed zones. The reduction in area by one-half accounts for 3-db loss while the variation in phase over exposed zones accounts for an additional 2-db loss, a total of 5-db loss for a zone plate antenna compared to a parabolic reflector of the same diameter. A parabolic reflector has the property of transforming a plane wave into a spherical wave with center at the focal point of the parabola.

[6] G. F. Hull, Jr., *Am. J. Physics* **15**, 111 (1947).

Fraunhofer diffraction from a rectangular or circular opening is exhibited by the radiation patterns of a rectangular or circular horn, or a parabolic reflector.[6] To demonstrate or measure a Fraunhofer diffraction pattern, a horn with receiver attached should be mounted on a rotating stand at a large distance from the microwave transmitter. The transmitter and horn whose pattern is to be measured should first be lined up and then as the horn is rotated, the received microwave power as a function of angle is measured. The vacuum tube voltmeter connected to the crystal detector of the receiver will measure relative microwave power directly provided the crystal obeys the square law. To obtain the diffraction pattern of a parabolic reflector, the receiver should be mounted at the focus of the parabola and the entire assembly rotated about a vertical axis as is done with the horn.

Another interesting demonstration, as well as an instrument for precision measurement, is the microwave Michelson interferometer. This instrument is shown in Fig. 5 and consists of the microwave transmitter, two totally reflecting mirrors of brass 10 in. square, mounted on movable supports on the wooden benches, a half-reflecting mirror made by winding wires $\frac{1}{2}$ in. apart on a 12-in. plywood board, which is mounted on a rotating support on a wooden bench, and the receiver which is shown clamped in a rod stand in the foreground. The transmitter, mirrors, and receiver must be carefully lined up. When one of the totally reflecting mirrors is moved slowly along the wooden bench, the receiver will indicate the passage of maxima and minima corresponding to bright and dark fringes in the optical case. In fact, when the interferometer is properly adjusted, the minima are extremely sharp and are 30 to 40 db below the maxima in intensity. If several sheets of dielectric such as glass or plywood are placed in one arm of the interferometer, the receiver will indicate maxima and minima as the sheets are removed one after the other. The index of refraction of a dielectric can be measured by noting the fringe shift produced by the insertion of a known thickness of dielectric in one of the interferometer arms, as is done in the optical case. In the microwave region the index of refraction of ordinary window glass is about 2.0 and that of plywood

about 1.3. Hence, four sheets of single weight window glass or four sheets of $\frac{1}{4}$-in. plywood inserted in one arm of the interferometer will produce a shift of about one fringe. The microwave Michelson interferometer is capable of high precision and is especially useful in measuring the dielectric constants of artificial microwave dielectrics which will be discussed in the next section and which cannot be placed inside a 3-cm wave guide. As a precision instrument, the interferometer must be rigidly constructed with rigid mirrors equipped with screw drives.

Refraction and Artificial Dielectrics

The refraction of microwaves by dielectric materials can be demonstrated in many ways. In the case of microwaves it is possible to use ordinary matter as the refractive medium which may or may not be transparent to light, or because of the special properties of microwaves, artificial dielectrics can be constructed which refract microwaves but not light. Two types of artificial dielectrics will be discussed.

In Fig. 6 two 60° prisms, 10 in. on a side, are shown. The prism on the left is made of paraffin (index of refraction 1.47) and the one on the right is made of sheets of galvanized iron forming parallel-plate wave guides. The paraffin prism is contained in its form which is made of $\frac{1}{4}$-in. plywood. If the paraffin prism is placed in front of the horn radiator, it is found that the microwave beam is bent in the same way that a light beam is bent by a glass prism. Holding the microwave receiver in the hand and rotating the prism, one can locate the angle of minimum deviation, which for a 60° paraffin prism is 33°. The wave guide prism on the right in Fig. 6 operates only as a prism when the electric vector is parallel to the metal plates of the prism. For this case the TE_{01} mode for a parallel plate wave guide is excited, and the wave velocity in the parallel plate guide is greater than the free space velocity. The wave velocity is given by $v=c[1-(\lambda/2b)^2]^{-\frac{1}{2}}$, where c is the velocity of light, λ the free-space wavelength, and b the spacing between the metal plates.[2] The index of refraction of the parallel plate wave guide is then $n=[1-(\lambda/2b)^2]^{\frac{1}{2}}$. The prism shown has an index of refraction of 0.6 corresponding to a plate spacing $b=0.79$ in. The plate spacing is critical and should be maintained to within ± 2 percent. Because the index of refraction is less than unity, the microwave beam is bent in the opposite direction from what it is for the paraffin prism. By rotating the wave guide prism, an angle of minimum deviation can be located with the aid of the receiver and is $-24°$. If the wave guide prism is oriented with its plates perpendicular to the electric vector, the TM_{00} mode is excited whose wave velocity is the same as in free space,[2] and consequently the microwave beam is not deviated. The artificial wave guide dielectric prism therefore operates only for plane-polarized microwaves with the electric vector parallel to the wave guide plates. Evidently this dielectric will exhibit a type of double refraction for unpolarized microwaves, and later we shall describe experiments in which this property of the metal plate dielectric is used.

Besides prisms, lenses can also be constructed. Plano-convex lenses of paraffin can easily be made by filling 8- or 10-in. diameter watch glasses with paraffin. Glass lenses and lenses made of artificial dielectrics can also be used. In Fig. 7 three lenses are shown. The lens at the left is a 10-in. diameter glass, plano-convex condensing lens, the lens in the center is an artificial dielectric lens made up of an array of thumb tacks, and the one on the right is a lens made up of parallel-plate wave guides. All of these lenses exhibit the usual properties expected of lenses. Their focal length for 3-cm microwaves can be determined experimentally within $\pm\lambda$, which, considering that the lens diameters are about 8λ, is reasonable precision.

The parallel-plate wave guide lens has the same refractive properties as has the wave guide prism previously described. The galvanized iron lens plates are supported in a wooden frame 12×12 in. with a plate spacing $b=0.79$ in. ± 2 percent which gives an index of refraction of 0.6.

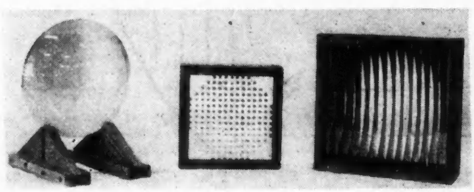

FIG. 7. Three lenses for 3-cm microwaves. Left to right are a glass lens, an artificial disk dielectric lens, and a wave guide lens.

Since the index of refraction is less than unity, a converging lens is plano-concave. Such a lens has a focal length of 12λ (15 in.), a diameter of 9λ, and a radius of curvature of 4.8λ. With these particular dimensions, it is not necessary to zone or step the lens, and the departure of the spherical surface from the true ellipsoidal surface is not greater than 0.2 in. or a phase difference of $\lambda/16$ at the extreme, which is within the tolerance limits for this type of lens.[7] To obtain the correct radius for each plate, it is simpler to draw the entire lens to scale and take off the radii with dividers rather than calculate each radius individually. Like the wave guide prism, the wave guide lens operates as a lens only when the electric vector is oriented parallel with the plates.

It is interesting to note at this point that we can define an index of refraction for a parallel-plate wave guide from which we can make the same type of calculations that is done for an ordinary dielectric. For example, we can calculate the reflection coefficient. Also, we can define a characteristic wave impedance for the parallel-plate wave guide and for free space, and from these quantities calculate the reflection coefficient. Both methods must yield the same value for the reflection coefficient, and consequently one would

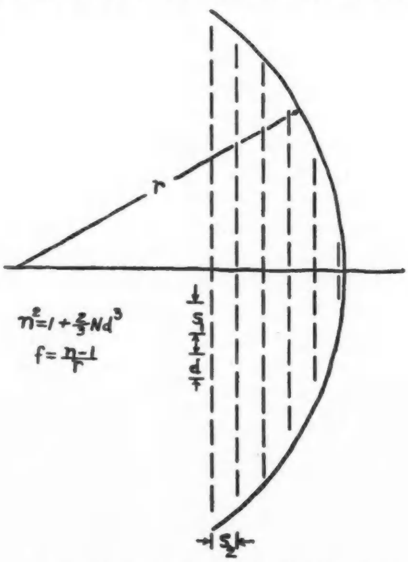

FIG. 8. Arrangements of disks in artificial disk dielectric lens for 3-cm microwaves. $n^2 = 1 + \frac{2}{3}Nd^3$, $f = (n-1)/r$.

[7] W. E. Kock, *Proc. I.R.E.* 34, 828 (1946).

expect analogs to exist between transmission line theory and optics.[8] Another example of this analog is the case of a quarter-wave transmission line and coated lens. Two transmission lines of different characteristic impedances can be connected, without producing reflection, by a third transmission line, a quarter-wavelength long whose characteristic impedance is the geometric mean of the characteristic impedances of the two lines. Similarly, a glass lens coated with a quarter-wavelength thickness of dielectric whose index of refraction is the geometric mean of the indices of refraction of glass and free space will be reflectionless for one particular wavelength.

Another interesting analog is the microwave equivalent of molecular arrays which is exemplified by the lens made of an array of thumb tacks shown in the center of Fig. 7. Since a piece of metal whose dimensions are small compared to a wavelength can be driven in forced oscillation by an electromagnetic radiation field, an array of identical metal pieces such as spheres, disks, or rods should behave in the same way that a dielectric made up of a molecular array behaves when exposed to light. In other words, an array of disks should have a dielectric constant $\kappa_e = 1 + N\alpha/\epsilon_0$ where α is the polarizability of the disk, N the number of disks per unit volume, and ϵ_0 the electric inductive capacity of free space. This is the same as the classical expression for a dielectric in which α is the polarizability of a molecule and N the number of molecules per unit volume.

The expression for κ_e is applicable only when the disks are far from resonance and there is no interaction between the fields of adjacent disks. In general, we would expect an artificial disk dielectric to obey the Clausius-Mosotti equation at long wavelengths and to exhibit the phenomenon of anomalous dispersion at short wavelengths when the microwave frequency approaches the resonant frequency of the metal disks.

The general criterion for the design of an artificial dielectric made up of an array of identical metal elements is that the dimensions of the elements should be less than $\lambda/4$ and the spacing of the elements less than λ. If the spacing is greater than λ, diffraction occurs similar to x-ray diffraction by crystals. Furthermore, the metal elements should be thin in the direction of

[8] Ref. 2, p. 95.

propagation of the microwaves.[9] On the basis of this criterion, the artificial disk dielectric lens shown in the center of Fig. 7 was constructed. Since the polarizability of a metal disk is $\frac{2}{3}\epsilon_0 d^3$, where d is the disk diameter, the dielectric constant or square of the index of refraction of a disk dielectric is $n^2=\kappa_e=1+\frac{2}{3}Nd^3$. The actual array used is shown in Fig. 8 and the dimensions shown are $d=1$ cm, $S_1=1.3$ cm, and $S_2=1$ cm. These dimensions give $N=1.18$ disks per cm^3 and a calculated index of refraction $n=1.33$. The metal disks used were thumb tacks $\frac{3}{8}$ in. (0.95 cm) in diameter which were stuck into sheets of polystyrene foam 1 cm thick and 8 in. square. Polystyrene foam has a density of about 1.5 lb per ft^3 and an index of refraction of 1.01. It has practically no refractive effect on the microwaves. The lens is plano-convex with a radius of curvature of 5 in. and a calculated focal length of 15 in. Five sheets of polystyrene foam are used. As shown in Fig. 8, the disks in alternate layers are staggered. This is done in order to increase the number of disks per cm.[3] To determine the radius of the circular area to be covered by thumb tacks on each sheet of foam, it is simpler to draw the lens to scale and take off the radii with dividers rather than calculate each radius individually. The positioning of the thumb tacks is best accomplished by marking out the circular area and dotting in the thumb tack centers on thin paper. This paper is then placed on the foam sheet and the thumb tacks pushed through the paper into the foam. After all the sheets have been filled with the required number of tacks, the sheets are put together and supported in a wooden frame. The lens when completed and measured is found to have a focal length of 10 in. instead of the calculated value. This means the index of refraction of the disk dielectric is 1.5 instead of the calculated value of 1.33. The discrepancy between calculated and measured index of refraction is to be expected because of the effect of the Clausius-Mosotti equation and because the diameter of the thumb tacks is slightly larger than $\lambda/4$. Unlike the wave guide dielectric lens, the disk dielectric lens operates independently of the polarization of the microwave radiation. Also the index of refraction of the disk dielectric remains essentially constant for longer wavelength microwaves, which is not the case for the wave guide dielectric.

[9] W. E. Kock, *Bell. Sys. Tech. J.* **27**, 58 (1948).

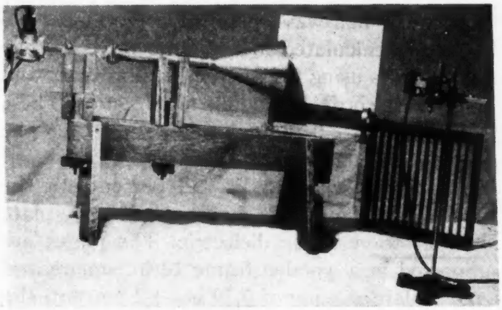

FIG. 9. Transmitter and receiver arranged for producing plane-polarized, 3-cm microwaves by reflection from glass at the Brewster angle.

The index of refraction of artificial dielectrics can be measured with high precision with the microwave Michelson interferometer discussed in the previous section. The procedure is the same as in optics. Essentially what is done is to insert a sheet of artificial dielectric about one foot square and of known thickness in one arm of the interferometer and measure the fringe shift, from which the index of refraction can be calculated.

Polarization

As has been pointed out, the microwave radiation from the transmitter is plane polarized. For polarization experiments it is often desirable to have elliptically or circularly polarized radiation. Elliptically polarized radiation is easily obtained by placing a sheet of glass, polystyrene, or other dielectric in front of the horn radiator with the plane of the sheet at 45° to the electric vector and parallel to the direction of propagation. Elliptically and circularly polarized microwaves can also be obtained by use of the artificial wave guide dielectric discussed in the previous section. If this dielectric is made with a plate spacing to give an index of refraction of 0.6 when the electric vector is oriented parallel to the plates, it will also have an index of refraction of unity when the electric vector is at right angles to the plates. Consequently, if an appropriate thickness of wave guide dielectric is placed in front of the horn radiator with the plates oriented at 45° to the electric vector, elliptically-, circularly-, or plane-polarized radiation will result. The thicknesses for a

quarter- or half-wave plate of wave guide dielectric are calculated in the same wave as they are in optics using 0.6 for the extraordinary and unity for the ordinary indices of refraction. These thicknesses are 0.78 in. and 1.58 in. for the quarter- and half-wave plates, respectively, and the thickness tolerance is ± 2 percent. In the background of Fig. 9 is shown a half-wave plate made of wave guide dielectric. The plates are supported in a wooden frame 12 in. square and have a plate spacing of 0.79 in. ± 2 percent, the same spacing as the wave guide prism and lens. Although the quarter- and half-wave plates behave in a manner similar to those used in optics, the wave guide dielectric is not exactly similar in its double refracting properties to a uniaxial crystal, because the wave guide dielectric does not have an optic axis. In general, microwaves pass through the wave guide dielectric with two components one of which travels faster than the other and whose amplitudes depend upon the orientation of the electric vector of the incident microwave radiation with respect to the plates forming the wave guides.

One can extend the principle of the quarter-wave plate to a circular wave guide so that the radiation from a circular horn will be circularly polarized. All that is necessary to do is to split up the microwave radiation in a circular wave guide operating in the TE_{11} mode into two components of equal amplitudes and at right angles and delay the phase of one component by $\lambda/4$ with respect to the other. This can be achieved by inserting in a circular piece of wave guide a sheet of dielectric along the diameter and oriented at 45° to the electric vector of the incident microwaves. The proper thickness and length of such a dielectric is not easily calculated, but can be found experimentally. The horn radiator shown in Fig. 9 attached to the transmitter has a polystyrene strip $\frac{1}{16}$ in. thick and 2.5 in. long across the diameter of the 1-in. standard circular wave guide leading to the horn. The strip is oriented at 45° to the electric vector of the incident microwaves and the horn radiates circularly polarized radiation. If this polystyrene strip is replaced by one of the same thickness and 5 in. long, the result is a half-wave plate and the radiation coming from the horn is plane polarized with the electric vector rotated 90°.

A number of interesting experiments can be performed with various types of polarized microwaves. Circularly polarized microwaves can be plane polarized by reflection from a dielectric at the Brewster angle. In Fig. 9 the circular horn equipped with a quarter-wave plate radiates circularly polarized microwaves which are incident upon a sheet of window glass held in a rotating support on the wooden bench. The reflected microwaves are investigated with the receiver which is also an analyzer and when the glass is at the Brewster angle the reflected microwave radiation is plane polarized. The index of refraction of dielectrics can be measured by this method in the same way that it can be measured at light wavelengths. It is interesting to note that the thickness of window glass is about $\lambda/16$ for 3-cm microwaves, and is therefore a thin film as far as microwaves are concerned. Evidently this technique can be extended to measure the index of refraction of thin films less than a wavelength thick in the region of light wavelengths, which is difficult by other methods. Probably the thin film less than a wavelength of light thick would have to be mounted on a thick backing material, in which case the Brewster angle for the thin film and the backing dielectric would both be obtained.

Experiments on the rotation of the plane of polarization of microwaves by sugar solutions, liquids such as turpentine, and crystals such as quartz have not shown any measurable rotation. However, it has been found that the Faraday effect exists in the microwave region for certain paramagnetic salts.[10] Finally one might ask if double refraction exhibited by crystals in the region of light wavelengths also exists in the microwave region. The author has found that a Nicol prism will polarize microwaves as well as light, but Polaroids will not. This means that the indices of refraction for the ordinary and extraordinary rays of calcite must be nearly the same for microwaves as for light. The Nicol prism used in the microwave region must be large, and the author uses one 6 in. long.

In conclusion the author wishes to thank Mr. W. Durrschmidt and Mr. E. Fitzgerald who helped construct the microwave apparatus.

[10] M. C. Wilson and G. F. Hull, Jr., *Physical Rev.* **74**, 711 (1948).

A Summer Refresher Program for High School Physics Teachers

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TWO years ago representatives of the General Electric Company approached the writer to inquire whether Case Institute of Technology would undertake the further training of a selected group of high school teachers in a broad effort to improve the teaching of physics in American high schools. As one of the largest employers of scientists in this country, the General Electric Company has a very real interest in the better training of American scientists. In considering how best the Company could influence this training, those in charge of its educational program decided that, since the habits of precise and logical thinking needed in science are acquired quite early in a student's training, the most effective place to begin would be in high school, and the most effective way to reach these students would be through their high school teachers of physics and chemistry.

Deficiencies in the high school training of young scientists are most likely to become apparent first in college, and the Company, therefore, decided to request two strategically located colleges, Union College[1] in Schenectady, and Case Institute in Cleveland, to make proposals as to the type of program they would consider most useful for high school teachers, if offered during a six-weeks period in the summer. The Company, on its part, offered to provide fifty fellowships, paying tuition, travel, living and other expenses of the teachers for the six-weeks period. It was suggested to Case that all the candidates be chosen from among teachers of physics and that they be selected from the central states, extending from Western Pennsylvania to Indiana, Illinois, and Wisconsin.

This request was indeed a challenge to the administration and physics faculty at Case. As at most colleges, Case faculty members are seldom satisfied that the training their students have received in high school is the best possible. Now they were to be given a golden opportunity of saying just what was wrong with this training, and what they would do about making it better. After many discussions, a three-part program was proposed, consisting of organized courses on: (1) Basic Concepts in Physics, (2) Frontier Problems in Physics, and (3) Application of the Principles of Physics in the Control of Environment. This program was to be under the general direction of the writer, but with the able assistance of Professors R. S. Shankland, R. C. Putnam, L. O. Olsen, and E. C. Gregg.

It was, of course, realized that when fifty teachers of high school physics broke their home ties to work, eat, and live together for six weeks, they would learn more from each other than from the formal courses. The prime purpose of the courses and of the college professors would be to direct their thinking and to set high standards for them to follow.

The program suggested was approved by the General Electric Company and the fellowship program was established at Case for the summer of 1947, repeated during 1948 and was offered a third time in 1949. The first problem in getting the program under way was that of adequate publicity so that the best possible candidates could be reached. It was necessary, as far as possible, to reach not only the high schools in the large cities, but also those in smaller communities where there is undoubtedly much potential scientific talent. Through the public relations officers of the General Electric Company at Nela Park and those at Case, a list of some 7000 high school principals in the states concerned was obtained. A two-page folder describing the program was then sent out with the hope, which in some cases was in vain, that the principal would turn it over to the physics teacher. Releases also were prepared for newspapers in the regions concerned and for scientific and teaching periodicals. As a result, a good response was obtained and about 200 applications were received each summer. The most important deterrent to more applications was that high school teachers nowadays depend greatly upon what they can earn during the

[1] The program at Union College, under Professor V. Rojansky, was started in 1945, two years before that at Case.

summer, and they often cannot afford to accept a fellowship even though all expenses are paid.

The fellowship winners were chosen by a committee from Case taking into account many factors. For example, considerable weight was placed on the academic records in the sciences of the applicants, since it was likely that those who had been most successful in science in college would be most likely to encourage their pupils to study science. Also, younger teachers rather than those nearing the retiring age were chosen, so that the benefits of the program would extend over as long a period as possible. It seemed advisable to make the geographical distribution over the states concerned as uniform as possible, consistent with high standards, especially after it was realized that fellows, upon returning home after the completion of the program, were themselves excellent emissaries for good teaching of physics over a wide region.

The successful applicants in 1947 included 45 men and 5 women, with an average age of 40; and in 1948, 46 men and 4 women, with an average age of 39. Geographically, the distribution was as follows:

	1947	1948
Illinois	9	14
Indiana	8	7
Kentucky	2	0
Maryland	3	0
Michigan	8	4
Ohio	17	8
Western Pennsylvania	1	3
West Virginia	1	1
Wisconsin	1	13

After the awards were made, wide publicity was given to the winners, particularly in their home town papers. While this was done originally merely as a matter of good public relations, it was found that this publicity paid many extra dividends in the added recognition the fellows received in their own communities. One teacher said that previously few people in his community had any idea that physics was being taught, but after the awards were made, this particular winner found that he had become a very important person, and that everyone was remarking on the great value of good science teaching in that community. The regeneration of this teacher's deflated ego, as a result of the award, was of as much value to him as anything he learned in the course.

All in all, the fellows arrived in a very enthusiastic mood, and the program started auspiciously with a Sunday evening reception. The first formal class began promptly at nine o'clock the following Monday morning.

Since the courses offered are somewhat unique in a college curriculum, they will be described in some detail. The first course, entitled "Basic Concepts," was designed to meet the criticism which many college teachers have expressed that high school courses are too superficial; that students are taught too little of too many different subjects. An attempt was made, therefore, to present a very few important concepts in a thorough manner. Emphasis was placed on the basic achievement of modern science—the unification of a great variety of miscellaneous detail into a few important generalizations. It is believed that if students know these few principles well and realize that in knowing them they have a good grasp of the entire field of physics, they will be well trained. To point up even more sharply the idea of learning a few concepts well, one week was devoted to each of six important physical principles having wide applicability. For instance, the first week was devoted to Newton's Laws of Motion, and began with a discussion of the historical setting in which the *Principia* was written and with a brief study of Newton's unusual personal characteristics.

Great care was taken in the formulation of Newton's Laws to insure precise expression. No physical quantity was ever expressed in numbers without adding the necessary units. Problems were assigned in considerable numbers and it was insisted that the units of the answer be checked dimensionally with those placed in equations. In spite of many years of teaching, some of the fellows understood for the first time the directions of the forces acting upon a stone swung in a circular path at the end of a piece of string. They understood also the need for critical examination of the forces and the bodies upon which they are acting in the statement of Newton's Third Law. A systematic method of attack for Atwood's Machine type of problems was developed so that the fellows lost all fear of even the most complex problem of this kind.

To cover Newton's Laws thoroughly in a week meant a fast pace and hard work, but all agreed that it was time well spent. There was time on the last day of the week to bring out the fact that Newton's Laws are merely special cases of more general laws and that relativistic mechanics are needed whenever very high speeds are involved and that wave mechanics must be used to describe the behavior of very small masses.

There is not space to describe in detail the work of the remaining weeks in this course under the direction of Dr. Olsen. One week was devoted to the conservation laws, another to the laws of thermodynamics, still another to a simple formulation of Maxwell's Laws, and so on. After finishing this course these teachers had a new appreciation for the fundamentals of physics and a new determination to give their pupils the kind of high school training that is needed to make this country strong scientifically. Over and over, the question was asked, "Why didn't we learn to be more critical of physical concepts in our numerous courses in education?"

The second course of the series, that on "Frontier Problems in Physics," under the direction of Dr. Shankland, Head of the Physics Department, was designed to bring the high school teachers abreast of the most recent developments in atomic and nuclear physics. Fortunately, we were able to call upon many of those responsible for recent developments to describe their own work. Drs. William D. Coolidge, Saul Dushman, and W. F. Westendorf, and others, came from Schenectady to present outstanding reports of new work in the fields of x-rays, high vacuum techniques, and nuclear particle accelerators. Drs. C. G. Found, H. C. Froelich, N. L. Oleson, and many others from the Lamp Development Laboratory in Cleveland gave unstintingly of their time. There was always a sense of tense excitement in this course, and the high school teachers were greatly stimulated by these discussions of modern problems. There was little question but that they would be better able to follow the reports of new work in the everyday press, and to answer with intelligence and enthusiasm the questions asked by their pupils.

An important corollary to this course was a program of semiweekly demonstration laboratory periods under Dr. Gregg, in which Geiger counters and Wilson cloud-chambers could be seen in use and where students could become really familiar with x-ray equipment, the mass spectrograph, and the betatron. It was felt that contact with these modern tools, even though on a group basis, was more useful than attempting to provide the usual kind of elementary laboratory, and was something which the fellows probably could not duplicate in their own laboratories.

The third course, entitled "Science and Technology in the Control of Environment," under Professor Putnam, was something of an experiment, but one that offered many dividends. Its purpose was to point the way in which applied science enters every nook of our daily life in providing for our comfort and in enabling man to take greater advantage of his leisure time. A prime example of this, and one that ties in particularly with a program in Cleveland, is the extension of man's daylight hours through modern developments in lighting. Lectures on the application of physics to fundamental research on visible radiation, the development of new light sources, and the principles and practice of using radiation in the visible, ultraviolet and infra-red regions for the benefit of man were given by several scientists and engineers from the Lamp Department of the General Electric Company who are outstanding authorities in those fields. Other examples of the application of science are the control of air temperature, humidity, dust and bacteria content through air conditioning, and the control of noise through a better understanding of acoustics and the development of better absorbing materials. The lectures on air conditioning were presented by Cyril Tasker, Director of the laboratory of the American Society of Heating and Ventilating Engineers located in Cleveland. Even the control of weather came in for discussion and Vincent J. Schaeffer of the G.E. Research Laboratory in Schenectady gave an extremely interesting presentation of his recent work on "rain making."

The fellows had an unusual opportunity to observe and study ways in which physics is used in industry. Two afternoons a week were given to this program in which the fellows visited the Lighting Institute at Nela Park, selected factories, and the research and development

laboratories of the General Electric Company, the National Advisory Committee for Aeronautics, and the American Society of Heating and Ventilating Engineers. Groups were kept small, so that the scientists and engineers could outline their problems, show the fellows what was being done and discuss personally the specific phases of their work.

Enough has been said of the curricular activities. What about extra-curricular ones? Every effort was made to encourage group discussion. The fellows were assigned two or four to a room in near-by fraternity houses, and were given all their meals as a group. The instructors in the classes and the distinguished visitors were encouraged to eat with the fellows. An activities director assisted the fellows in arranging baseball, volley ball, ping pong and swimming contests. Thus the fellows had full days. On week ends many special trips were arranged, such as an all-day picnic excursion to the Mohican State Forest, where Case maintains a surveying camp; a visit to Cain Park, an outdoor summer theater; a conducted tour of the Cleveland Museum of Art; and even a trip to watch the Cleveland Indians win one of their ball games. On several afternoons, the complete facilities of the Nela Camp were made available, to be followed by an excellent outdoor dinner and talk by an outstanding scientist or Company executive.

There were usually about six hours of class and laboratory on each of the five week-days, with about two hours relaxation, and an evening in solving problems and studying the assignment for the next day. After only a few days of a schedule like this, reserve was broken down and we found teachers from Kentucky and those from Wisconsin discussing everything from a laboratory experiment to illustrate Newton's Laws to teaching loads and salaries. The climax of the summer activities was a dinner arranged completely by the fellows, with a program of their own very considerable talent, with the Case staff taking part only to the extent of passing out certificates to those completing the six weeks work.

Was the program a success? Were its objectives met? These are important questions which may not be completely answered for many years. Certainly, in terms of the enthusiasm of the fellows for better teaching at the end of the six-week period, the objectives were met. All the fellows went back to their classrooms with renewed interest in modern science, and with better understanding of the kind of preparation that should be given to high school students who are going into science as a life work.

As a final word, we will quote from a letter recently received by the Secretary of the General Electric Company from one of the fellows:

"May I introduce myself as one of the teachers on a General Electric Fellowship at Case Institute of Technology and Nela Park last summer? For a year I have desired to express my thanks to the Company for the privilege of being a member of the group which studied in Cleveland, but I have scarcely known to whom I should write. Since I have received several communications from you during the past year, I now address this letter to you.

"To meet a selected group of teachers in one's own field of interest from nine different states, and to study and live with them on intimate terms for a period of six weeks is a privilege. To have as teachers members of the excellent faculty of Case, and men of considerable reputation in their respective fields from General Electric and other institutions; to be in classes arranged especially for the group; and to be given an insight into the activities of a large Company like General Electric makes the privilege rare. To be treated as guests; to be well-housed, well-fed, and well-entertained; and to be given opportunities to grow acquainted with, and become friends of, our instructors and hosts from Case and General Electric is an honor. All of these things were made possible for us through the thoughtfulness and generosity of the General Electric Company. The experience will linger in my memory as one of the important events of my life, and the certificate received at the close of the sessions will be as highly valued as my college diplomas. I wish to extend my sincere thanks to the General Electric Company, and particularly to those individuals responsible for inaugurating and carrying out the Science Fellowship Program in which I participated. I hope that I can do my part in transferring the benefits of the experience to my students."

Important principles may and must be flexible.—Abraham Lincoln.

Teacher Training in the Graduate School*

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It is a very pleasant occasion for a psychologist to be invited to present his views to representatives of that most respectable of sciences, physics. My general intention is to lay the groundwork for suggestions for the improvement of college teaching, then to sketch a plan for producing such improvement, and finally to indicate some of the problems encountered in establishing a program and making it work. I should perhaps clarify my position by indicating that I am addressing myself to university faculty members more than to college faculties, and that I believe it desirable to proceed with such training as can be given now, rather than to wait to determine what more final form our educational system and policies should take. As a result of this latter belief, I am in effect assuming that teachers should be trained for teaching situations of the kind we now have, with the skills now useful. This does not deny the importance of discussions of educational philosophy. Rather, it supplements any such emphasis.

What Should a Teacher Learn?[1]

There are many essays on the knowledge and skills which should be possessed by a teacher, but as yet there is little objective evidence to support any essay. Nevertheless I shall list my ideas about essential teacher skills, further to indicate what I believe we must secure by training. The order in which ideas are presented is not intended to show relative importance, nor the amount of time which might be spent on them in a training program.

(*1*) *Purposes and nature of higher education.* Although I have stressed above the necessity for learning what is practically useful now, I should like to claim that some knowledge about higher education as such is an essential to doing well as a faculty member. The prospective faculty member should have thought about, or studied, questions concerning the relation of the college to other aspects of our culture, and the types of development which education is supposed to foster in our youth; he should have become familiar in some degree with the nature of general education, types of institutions in which he might take a position, etc.

(*2*) *Personal relations, with faculty, administration, students.* To a certain extent the success and well-being of a faculty member depends upon getting along in an understanding way with the other people who inhabit his academic world. Functions and responsibilities of department heads, deans, or academic committees are not so specific to a given institution that the prospective teacher may not learn in general what to expect, and thereby avoid making any serious blunders which might make him insecure or unhappy in his first teaching job. The peculiar relation of superior-but-equal, which the instructor must maintain with students, must be understood from the beginning. Dangers of political entanglements on a faculty, before one is ready for them, and relations with registrars, librarians, business offices and other dealers in red tape likewise ought to be understood early, with a realization that these persons or agencies also are essential in the American college.

(*3*) *Speech-voice effectiveness; lecture and discussion skills.* It is a truism that a teacher must be able to "put it across," yet far from all faculty members know the principles of effective speaking or discussion, or the devices by which communication may be made most effective. It is *not* sufficient to know what one wishes to say, although this is essential to saying anything well. As a by-product of skill at communication, might we not also hope here for improvement of papers at professional meetings, speeches made in faculty meetings, or speeches such as this one?

(*4*) *Testing, measurement techniques.* Any instructor must evaluate the progress of his students, not only to satisfy the registrar but for pedagogical reasons. He should learn, therefore,

* Prepared from notes for a paper given at the Tenth Annual Colloquium of College Physicists, at the University of Iowa, June 17, 1949.

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[1] This section is comparable to a section in a paper entitled "On the Training of College Teachers," in press in the *Journal of Higher Education.*

what techniques are pertinent in his field, and perhaps more important, learn the principles according to which testing or measurement devices are developed, so that he may constantly improve the procedures available to him initially.

(*5*) *Visual aids, demonstrations, laboratory techniques.* Here is an area in which the physicist should set the pace among educators. Any new teacher should know how to create teaching aids, and understand the pedagogical principles involved in their use. Such aids may assist in teaching what cannot be taught so well in any other way; they provide variety, change of pace, interest. There is every reason to master their effective use.

(*6*) *Advising, counseling.* There is a constant stream of students through the average instructor's office. They seek help in their studies, choice of courses or a major subject, vocational guidance, and sometimes assistance on personal adjustment problems. The instructor in physics should not assume that guidance is the area of the psychologist, for the physicist in a college does it every day. The instructor should therefore learn how to get behind symptoms, or how to interpret symptoms; he should know what sort of information students ought to have in settling various types of problems; he should know when a problem is out of his province, i.e., when the student should be referred to one type of specialist or another, rather than given departmental time. It is important here that the instructor be a realist, and particularly that he see the student's performance in his course or in his office as taking place in one part of a broader academic-cultural setting, for students sometimes are conformists in the act of talking to an instructor, and sometimes they are not, and how to interpret this behavior is a daily problem.

(*7*) *Social psychology of the classroom.* You might expect me, as a researcher in learning and memory, as well as a psychologist, to state that every college physicist should master the laws of learning, in order to control the learning of his students. However, the laws of learning which are found in educational psychology textbooks, and which some of you have studied with a seemingly fruitless result, are probably less important than an understanding of a class as a social group. By this I mean that an understanding of how to motivate a student group, secure high working morale in it, use reward and recognition judiciously, capitalize on socially determined motives, will permit the instructor to control the most important single variable in learning: motivation. We must work with social motivation, and if motivation to learn, stemming from this source, is high, we have established a primary condition for learning. It is of secondary importance whether we use large assignments or small, massed practice or distributed, whole or part methods of study. Again, let me emphasize that for a college instructor to understand the motivation of his students he must see them, not only as present in his classroom, but as simultaneously part of a larger cultural and traditional setting. He cannot often go counter to culture, but he can capitalize upon it to produce attention to his subject matter.

Let me express one more qualification about the secondary laws of learning which I have treated so cavalierly above. They are *average* laws, i.e., they state what happens on the average under a standard set of laboratory or observational conditions. The application of these laws (the engineering use of them) depends upon stating the specific conditions under which they are to apply, i.e., the "settings" of all the relevant variables for a given course taught by a given individual to a given group of students. Since this is necessary, what we should strive for is a cooperative venture by, say, psychologists or educators, and physicists, so that physicists will finally be able to say something about the laws of learning of physics, under statable circumstances. This cannot be arrived at by nonphysicists who do not know the field, and it cannot economically be arrived at by physicists alone.

Inadequacies of Present Pre-Job Training

A good many instructors now take their first jobs without any previous teaching experience whatever. No one defends this, so far as I know, and attention is more often called to the number of instructors who have had experience as assistants while working toward the Ph.D. Even this experience is quite inadequate, to my way of thinking, for two general reasons.

(*1*) *The experience secured on assistantships is*

too limited, when available at all. (a) Assistantship duties are not only routine—they are only part of what a regular instructor must do. The total list of duties includes lecturing, demonstrations, laboratory work, examinations, conferring and counseling, tutoring, etc., and it is important that the interrelationships among these various activities be appreciated. What assistant has a chance to develop this appreciation? (b) An assistant has no reason to experience the full feeling of responsibility for the educational progress of a class. There is always someone else to take responsibility for disciplinary matters, decide on objectives and procedures, give the final word of advice. And one of the difficulties which some instructors exhibit is an inability to take responsibility, when they are actually on the job. The impact of the first job is, after all, rather heavy, for it is complicated, demanding, laborious; the person who is so insecure that he must consult other staff members constantly, or who cannot decide just how to handle things without a great deal of conflict, cannot enjoy teaching greatly or do a very good job at it. (c) Although I shall not harp on it, the content-experience of the Ph.D., that is, the concepts or courses which he masters, may be too narrow to prepare him well for teaching. Teaching is a game in which the unexpected demand is the common one, and only the broadest of backgrounds is helpful to the new instructor. (d) Because the assistant is neither fish nor fowl—neither a genuine faculty member nor merely a graduate student—he does not develop attitudes which are essential to easy faculty status. He is likely not to have a point of view on academic freedom, on the interrelationships of academic disciplines, or on the values of a life of teaching.

(*2*) *We do not provide sufficient competent guidance to the assistant who is learning about teaching.* (a) This springs partly from inadequate supervision. Usually, only the conscience of the staff member who has some course assistants will lead him to do anything more than make sure the routines of the course are running smoothly. The development of an assistant as a teacher is not rewarded by any kind of academic recognition for the staff member, and so he visits classes only occasionally, discusses the construction of examinations only to the essential minimum, and otherwise leaves the graduate student to learn for himself. And this is difficult, for he has only the clues which his students give him, to tell him how well things are going. Even the experienced teacher is hard put to it to interpret student reaction at times, so the assistant cannot be expected to profit much from it. (b) The inadequacy of guidance for assistants springs partly also from bias, bias on the part of the physicist or other specialist against the possible contributions of psychologists, educators, or speech experts. How many physicists send their students to take courses in the areas named? And is your reaction perchance: what good would it do? The fact is, of course, as I indicated above in talking about the laws of learning, that the direct offerings of, e.g., the psychologists, may *not* be of immediate and direct use to the physicist, yet many basic suggestions can come from such specialists. But again, what is needed is not suspicion or bias, but rather co-operative effort by physicist and methods specialist, so that the physicist is finally able to adapt for his prospective teacher whatever is of value in the offerings of nonphysicists.

Arguments About the Value of Formal Teacher-Training Plans

Having talked with a good many people about teacher-training plans, and having encountered various kinds and intensities of resistance to the idea, I think it in order to consider the arguments. But first, let me say that I do not think formalization a virtue. Rather, it is only and merely a device for *ensuring* training, and it does not imply to me an emphasis on forms without mastery of content.

(1) Thus far the most common objection to teacher-training has been that it will weaken the Ph.D. As I shall indicate below, this is not necessary at all. But further, people who make this objection seem to have their own stereotypes as to what the Ph.D. means, whereas in fact it has meant many things at different times, in different institutions, or in different subject-matter fields. Therefore, even if the meaning of the Ph.D. were changed somewhat by making some physics Ph.D.'s into teachers, I cannot see that any real harm has been done. (2) There comes next the argument that the informal

method of training is better than the formalized (and methods-centered) variety. Since I have already pointed out what I see as the weaknesses of assistantship training, I need only add that informality leaves too much to imitation and to random observation of the wrong things as well as the right ones. What physicist would stand for informally arranged experimental procedures? Then why stand for informal and haphazard training to teach? Further, it is not necessary that formalization of a program imply empty formalities taught to teachers. You, after all, determine the content of any training program you adopt, and you can make it your business to eliminate trifling matters according to your own standards. (3) It is true that promotion in rank or salary often do not follow upon good teaching, or upon production of good teachers, and so the objection to training programs is made that the best physicists or psychologists will not contribute to them. As a realistic proposition, at the present time, it seems to me that the best people can be persuaded to help train teachers only if they are permitted to retain their normal devices for securing promotion. Specifically they must be permitted their present research time, and time contributed to a training program should come off the remainder of their teaching load.

There are many other arguments[2] but the most stubborn ones have been presented above. At present, my reaction is: no substantial arguments (except inertia) *against* the training of prospective teachers have been advanced, and therefore the question to be faced is: do we, or do we not, value good teaching as much as other specialities within our respective professional areas? If we value it highly, we should try out a training program and see whether it does what we want.

It is clear that certain assumptions have to be made by anyone who is realistic about proposing a program at the present time and these are, at a minimum: little change in the existing program of a department, and in its relations with other segments of the university, such as the college administration or the graduate school; an interest in co-operation, locally, between subject-matter specialist and other specialists; some financial support, i.e., an administration convinced that teacher training is worth while.

Proposal of a Plan

My organizational plan for a teacher-training program is but an elaboration of the obvious. Its possible virtues lie in its ensuring that training will actually be given, rather than in any new conception of what ought to be done. (1) A few third-year Ph.D. candidates would be appointed as teaching assistants in each interested department, as they now are, *but* these trainees would each be given *complete* responsibility for a section of an elementary course, to occupy in it, so far as the undergraduates are concerned, exactly the same status as would a full professor. (2) A departmental supervisor would be appointed in each interested department. He would be freed half-time from other teaching duties, but no limitations would be imposed upon his research time. His duties would include conducting a one-term seminar on teaching problems in his field, conducting planning sessions for the guidance of the trainees, visiting classes and conferring with the instructors afterward, and in general talking informally, evaluating, motivating, wherever his judgment indicated these were needed. If he has the time and the interest, he will create his own set of operations for producing improved teaching by the trainees. (3) A co-ordinator should be appointed, in the office of the graduate school, to integrate the training activities of the several interested departments, to work with the supervisors in any matter in which they request it, and to conduct a seminar concerning any matters common to all subject-matter areas (e.g., construction and use of examinations, nature of educational systems, grading systems, social psychology of the classroom). This seminar, of course, could make use of invited experts, and could be extended to include speech training although this might be arranged for independently.

It should be noted (1) that the plan suggested does not take any responsibility or authority away from the subject-matter department, thus leaving co-operation or persuasion as the basic

[2] I have discussed some of them in an article titled "The Pros and Cons of Training for Prospective College Teachers of Psychology," in press in the *American Psychologist.*

processes in any adoption of training plans. (2) The budgetary demands would include mainly the teaching load from which the departmental supervisor was freed (and this can often be absorbed elsewhere within the departmental load if desired strongly enough) and the cost of a co-ordinator. The largest single item, payment of a reasonable wage to the apprentice instructors, is likely to extend the present budget little or not at all, for nearly enough assistants are usually available in large elementary courses before a program is instituted. (3) The plan is flexible to the extent that if a university-wide scheme is not practicable, any department can set up its own program in which the departmental supervisor is the prime mover and conducts all the training offered.

To illustrate how a plan may develop in a specific case, let me describe briefly the system which operates in my own department at Northwestern University. I am the departmental supervisor and no other Northwestern department has such a program. So, (1) I offer a two-quarter seminar on teaching, the first quarter being entirely concerned with practical matters such as lecturing; setting up examinations or demonstrations; handling disciplinary or morale problems; correlating text, instructor and other elements of the course; comparative studies of various teaching techniques; the goals of teaching; counseling at the academic level; techniques for teaching scientific method; use of teaching aids; teacher relations with students and faculty; evaluation of teaching. The second quarter is concerned with broader, more theoretical questions about the nature of higher education and educational systems or movements in this country. (2) For the graduate-assistant instructors, there is a weekly planning session, in which we block out (merely) the plans for the next week, each man having responsibility for all decisions about exactly what to present, when and how; at this session will also occur discussions of examinations, teaching attempts which went wrong, possible teaching procedures, and anything else whatsoever which is pertinent to the job of teaching. Often it concludes with a more general and rambling discussion of various aspects of the life of teaching. (3) For one quarter the graduate-assistant instructors are visited in class by a paid "tutor" (staff-member) from the School of Speech, who later confers with the trainee about his performance. In addition, a few group sessions are held, wire recordings and heckling sessions utilized, and anything else which might be helpful is tried. (4) For one quarter, I visit about one class in five taught by a given man, and confer with him later in the day. In the second quarter, I visit about one class in eight, and drop out these visits as soon as an individual seems entirely able to get along without them. (When I visit the class, I remain as inconspicuous as possible, but take systematic check-sheet notes; both students and instructor adapt to my presence with surprising quickness—the students, in fact, having as yet given no sign of knowing I am present.)

Problems Encountered in Establishing a Training Plan

(1) The principal sore spot thus far is the question of whether academic credit should be given for learning how to teach. I argue that it should, in order that a quality-control is provided for the teaching, and in order to provide a permanent record of what a trainee did. In our own department, it turned out that the Ph.D. requirements permitted elective hours, and in effect we have substituted partly for these about 10 quarter hours of credit for a year's experience at teaching. Whether or not the credit is an essential feature of the program seems to depend upon the local atmosphere—if full and enthusiastic participation by trainees and staff is secured without it, there is no reason to make a fetish of credit. On the other hand, the giving of credit is in a sense the ultimate in recognition of an element in Ph.D. training, and as a symbol of recognition makes the whole program much more convincing.

(2) There may be some difficulty in freeing the departmental supervisor from other duties, or in finding someone to take on the supervisor's job at all. If a department is convinced of the desirability of it, the supervisor's time can probably be freed, but if no one is interested in training in a particular department, it would be folly to force establishment of a program. This is true especially because any supervisor will have,

really, to work out the principles he will follow as he goes along. This will be well done only where there is sustained interest.

(3) Care must be taken to ensure that favorable public relations are maintained. The quality of teaching done by the trainees must be sufficiently good that there is no justifiable complaint about cheating the undergraduates (in two institutions, I have evidence that hard-working trainees actually do somewhat better teaching than less strongly motivated but experienced senior staff members). The trainees, set down in our present educational system, have a fine line to toe—they are not of the faculty, but must in part act that way, yet they are not free and simple graduate students. They must see this position clearly, so to conduct themselves that the department, as well as they themselves, will be praised rather than scorned. (This of course is not a great deal different from the expectations we have of all faculty members.)

Evaluation

As you think over what has been said, I think you must agree with my earlier statement: few or none of the ideas expressed here is new to you, and the bulk of my argument is that by doing something to *plan* for training we can capitalize better on the facilities we already have. Whether the training occurs in the predoctoral years, or in a postdoctoral internship, or whether credit is given or not, are really just details to be settled to suit a local situation. The important considerations are to make learning to teach easier, make teaching better, and make teaching more rewarding from the beginning. As a final statement, let me point out for those who believe advancement should be based on scholarly productivity, rather than quality of teaching: if we can do anything which makes the first years of full-time teaching easier and more satisfying, we also make it easier for the new instructor to get his research and other scholarly activities rolling.

Some Recollections of Henry A. Rowland, 1848–1901*

HENRY CREW
Northwestern University, Evanston, Illinois

SINCE I am one of the few present who had a personal acquaintance with our first president, Henry A. Rowland, you will, perhaps, allow me a few words to say just how I happened upon this privilege. Paradoxical as it may appear, I think I can say most briefly what I want to say about Rowland by telling you just how I first chanced to meet him at a time when I was separated from him by more than 3000 miles.

The autumn of 1883 found me registered in the University of Berlin for a course in *Optics* with Kirchhoff, a course in *Experimental Physics* with Helmholtz and a course in *Spectroscopy* with H. Kayser. Like any other lad alone, and for the first time, in a foreign country, I took kindly to any citizen of my native land, regardless of race, color, religion, sex, or previous condition of servitude. Among the various Americans whom I chanced to meet in Berlin were James E. Keeler and D. B. Brace. Keeler had just finished the undergraduate course at Johns Hopkins; and Brace had just spent a year in Rowland's laboratory. Some of you will remember Keeler as the second director of the Lick Observatory: some will remember Brace as the man after whom the Brace Laboratory at the University of Nebraska is named.

Of the two, I knew Brace much more intimately. Early members of the American Physical Society will remember that he was in regular attendance at our meetings up to his untimely death in 1905. Naturally Brace's first question to me was "What are you doing in Berlin?" Having answered his query in some detail, it then became my turn to ask *him* how he came to leave Johns Hopkins University. He then told me (sometimes over the luncheon table in a near by *Rathskeller*, sometimes during a stroll

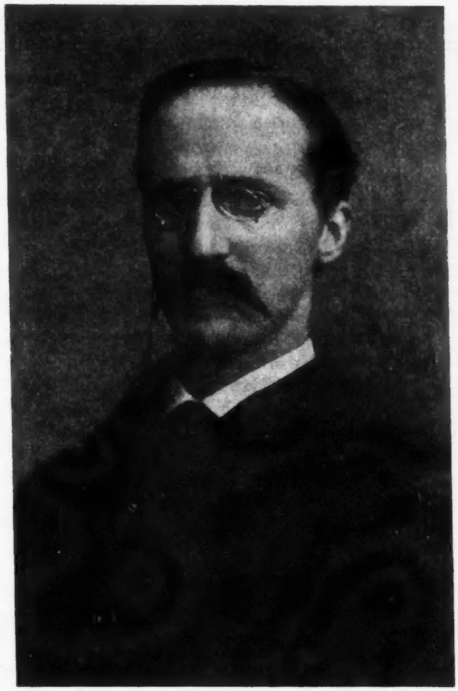
* Part of an after-dinner talk by Professor Henry Crew at the Semi-Centennial meeting of the American Physical Society, at Cambridge, Massachusetts, 17 June 1949.

in the *Tiergarten*) about the outstanding genius who was directing the work in physics at Baltimore; "but," he added, "I could never get along with him! He never understood me: and I, sometimes, did not understand him." He then went on to describe Rowland's marvelous experimental skill, his ability to concentrate on the work in hand, his forthright manner of speech, and his readiness to "cut" a lecture when necessary. And yet, in the same breath, after describing some of the unpleasant features at Baltimore, he said "Why! It is an inspiration just to see that man walk along the corridor in his fox-hunting boots!"

A little later he told me about Rowland's discovery of the magnetic analogue of Ohm's Law; and of his establishing this fact experimentally. All this, of course, before the introduction of such helpful terms as *magnetomotive force* or *magnetic reluctance*. At another time I listened to the story of Rowland's "Berlin Experiment," so-called, in which he established the fact that convection currents produce the same kind of magnetic field that other currents do, thus placing Maxwell's electromagnetic theory on an experimental basis. All this was done while he was still in his twenties. At another time Brace told me about Rowland's superbly accurate determination of the mechanical equivalent of heat and of his many new refinements in calorimetry and thermometry. Brace knew also about the ruling engine and Rowland's invention of the concave grating.

By this time, I had begun to ask *myself* what I was doing in Berlin, so far from rich opportunities in my own country. The upshot of these conversations with Brace was that, before the end of the semester, I was aboard a steamer for New York; and by the middle of April was at work in Baltimore as an assistant on an experiment which Rowland and Kimball were making for the United States Government. By the first of October I was a fully registered graduate student.

During the four years which followed I found Rowland to be all that Brace had pictured him to be and more—always clear, always frank, always modest. His two-year lecture course, given at that time in what was formerly an upstairs bedroom, covered the principal topics in heat, light and electricity. To attend his Journal Club was a rare privilege. Here was discussed the best work done in England, France, and Germany; Rowland's comments upon the work of other men were especially penetrating and stimulating and often surprising. No one could attend the meetings of this club without discovering that Rowland possessed the saving grace of humor, always on a high level, often fine and delicate. His praise was given only to encourage good work. His criticisms were made only as a stimulus to renewed effort.



Henry Augustus Rowland in 1884.

I would say that Rowland's outstanding trait was the high standard of work which he set for himself and all his friends. Experimental work must be of the first quality; and must be concerned only with questions of fundamental importance. The truth must be told clearly, simply and fearlessly. In my judgment, Henry A. Rowland was a leader whose memory the American Physical Society may always proudly and gratefully cherish.

NOTES AND DISCUSSION

A Null Method of Comparing a Capacity with a Resistance

HERBERT GOLDSTEIN
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THE comparison of capacity with resistance in terms of a frequency provides an instructive experiment for an intermediate laboratory course in electricity and magnetism. If an air condenser of suitable geometry is used, the value of the capacity in esu can also be measured directly from the dimensions. The experiment then affords a comparison between the esu and emu systems, yielding a value for the velocity of light.

The standard apparatus for this purpose has been the Maxwell bridge. In the hands of able experimenters, such as Rosa and Dorsey, the bridge is capable of extreme precision and has been used in fundamental standards work. But the method suffers from a number of disadvantages when applied to instruction. Fundamentally, the bridge measures the impedance of the capacity at the switching frequency employed. For suitable capacities the impedance is invariably quite high, and there are all the customary difficulties attendant on bridge measurements of high resistance. In addition, the Maxwell bridge, although similar to the Wheatstone bridge, is not quite the same; and the derivation of the complicated balance condition is both tedious and difficult to explain. One may attempt to choose the resistances so that the simple Wheatstone bridge condition is a good approximation. But then the ratios are such that the bridge is particularly insensitive.

A null method of comparing capacity and resistance has been devised which is easy to set up and simple to explain. So far as is known to the author, it has not been previously published. While the accuracy probably cannot be made high enough for standards work, it is amply sufficient to demonstrate the principles involved.

The basic circuit used is illustrated in Fig. 1. A motor-

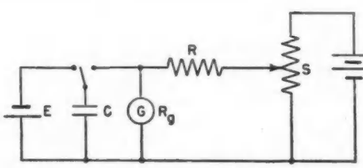

FIG. 1. Basic circuit for comparing a capacity with a resistance.

driven spdt switch alternately charges the condenser C to the potential E and discharges it through the galvanometer G. The effect of the resulting pulsating current on the long-period ballistic galvanometer is counterbalanced by a small steady current supplied by the slidewire S. The balance condition requires that

$$fCE=\frac{x_1V}{R+R_g},$$

or

$$C=\frac{V}{E}\frac{x_1}{f(R+R_g)},\tag{1}$$

where f is the frequency of switching, x_1 is the fraction tapped off the total voltage V across the slidewire, R is a series resistance, and R_g is the galvanometer resistance. The ratio E/V is determined by a separate potentiometer experiment using the same slidewire, as shown in the circuit diagram given in Fig. 2. If x_2 represents the fraction E/V in terms of a slidewire reading, then the final formula for C is

$$C=\frac{x_1}{x_2}\frac{1}{f(R+R_g)}.\tag{2}$$

The derivation of Eq. (2) has involved a number of simplifications. For example, it has been assumed the condenser discharges to zero potential. Actually it discharges through the galvanometer, across which there is a potential due to the small bucking current. However, the ratio of this potential to the condenser potential E is fR_gC, which is always completely negligible. A more consequential assumption is that $R+R_g$ is much larger than the slidewire resistance, but it is not difficult to choose the values such that this assumption introduces negligible error. Finally,

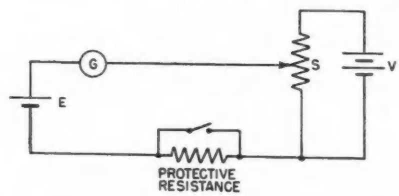

FIG. 2. Potentiometer circuit for determining ratio E/V.

R must be much larger than R_g in order that the discharge current flows mostly through the galvanometer and not through R and the slidewire. This condition is likewise easy to achieve. It is also necessary, of course, that the discharge time of the condenser be smaller than the switching period. However, because of the small capacities of air condensers and the mechanical limitations on practical switching rates, this requirement is automatically fulfilled with any reasonable set of circuit values. In our setup the discharge time constant is a matter of microseconds, while the switching period is 8 milliseconds.

In the particular arrangement of this experiment used in the Jefferson Physical Laboratory for the past three years, we have not attempted accuracies greater than one percent. Such an accuracy seems entirely adequate for pedagogical purposes. The switching is accomplished by a commutator resembling one-half of the Fleming and Clinton design. The central terminal of the switch is connected through a brush to a continuous slip ring, while the two outer terminals go to brushes riding on a slotted slip ring

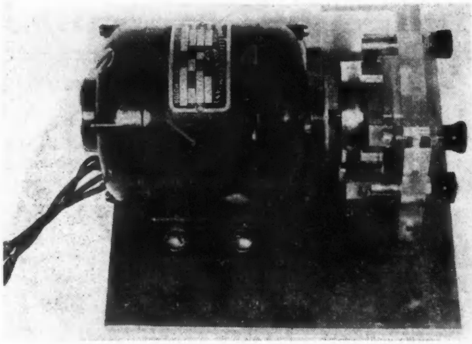

FIG. 3. Motor-driven switch. (Part of the protective Lucite cover has been removed.)

connected electrically to the continuous one. By adjusting the phasing of the brushes and the size of the gaps in the slip ring, it is arranged that one of the outer terminals is always unconnected while the other is in electrical contact with the center terminal. Details of the construction are shown in Fig. 3. The switch is driven by a synchronous motor fed from the local 60-cycle a.c. mains. Comparison with an accurately known frequency has shown that the line frequency rarely deviates by more than a few tenths of a cycle from its nominal value, which is sufficiently close for our purposes. A Strobotac is employed to show the students that the motor is running synchronously. To minimize the effects of distributed capacity of the switch and wiring a high capacity of 4000 $\mu\mu$f. is used, in the form of an old General Radio fixed air condenser (no longer manufactured). The plates are sufficiently closely spaced that fringing effects are small. The terminals of all circuit components are brought out to convenient binding posts so that the students may themselves connect up the two circuits needed. The cell E is an ordinary 1.5-volt dry cell and V is supplied by a heavy-duty, 2.1-volt storage battery. The ratio E/V is measured both before and after the main experiment and an average used to reduce the effect of any drifts.

It is clear that greater accuracy could be achieved, if desired. For example, one could measure the rotation rate of the motor directly instead of relying on its nominal value. In place of the slidewire, a regular potentiometer might be employed and the usual facility for checking against a standard cell used to minimize battery drifts. One can also attempt to measure the effects of distributed capacities, and higher charging voltages would increase sensitivity. For routine laboratory instruction, however, it seems more desirable to keep the experiment reasonably simple, so as not to conceal the principles in a maze of apparatus. One improvement that does appear to be highly advantageous is to replace the present condenser by an accurately constructed circular plate air condenser with guard rings, of the type used for secondary standards.

It is a pleasure to acknowledge the assistance of Mr. Carl Johnson of the Laboratory shop in the design and construction of the motor-driven switch.

Archimedes' Principle and the Hydrostatic Paradox—Simple Demonstrations

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WHAT happens to the water level in a lake when a metal ship sinks in that lake? Does it rise? Does it fall? Does it remain the same?

Is it possible for a 150-gram boat to float in 100 grams of water?

Try these questions in your more advanced classes in mechanics and be convinced that many students leave elementary physics without a clear understanding of Archimedes' principle. This may seem surprising since most students who have successfully passed through both high school and college courses in elementary physics seem to have little difficulty with problems on Archimedes' principle.

The following simple demonstrations, given a week or two after the conclusion of the study of the unit on hydrostatics, have proved very helpful in giving students a clearer understanding of both Archimedes' principle and the hydrostatic paradox.

The demonstration apparatus shown in Fig. 1 consists of a 1000-ml graduated cylinder C, a boat made out of an empty frozen orange juice can J, a three-inch brass wood screw B, and some lead or solder M. If the outside diameter of the can is not more than four or five millimeters less than the inside diameter of the graduated cylinder, the results will be most striking.

To make the boat, add pieces of lead or solder to the cleaned can to serve as ballast, until the combined weight of the can, lead, and screw is about 150 grams. Heat the can to melt the lead and make it adhere to the bottom of the can. Drill a hole in the center of the bottom of the can through the lead. The drill diameter should be just less

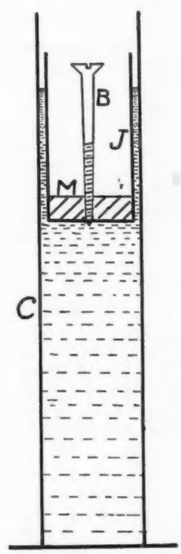

FIG. 1. Demonstration apparatus for Archimedes' principle.

than the root diameter of the screw used. The screw can now be made to thread itself in the lead and form a watertight plug. In addition to serving as a watertight plug, the long screw serves as a convenient device for leveling the boat. It also facilitates the handling of the boat.

Place about 800 ml of colored water in the graduated cylinder. Float the boat and note the water level. Remove the screw plug, place it inside the boat and allow the boat to sink. Interest will be added to the demonstration if the hole in the bottom of the boat is so small, that the boat takes a long time to sink. The water level in the graduated cylinder will remain unchanged while the boat is sinking, but will drop several centimeters as soon as the boat is completely submerged.

Quantitative results can easily be obtained. The boat is first "weighed" by noting the graduated cylinder readings before and after the boat is floated. The volume of the material of the boat is determined by noting the graduated cylinder reading after the boat has sunk. The average weight density and specific gravity can then be computed.

In the concluding demonstration, place 100 ml of water in the graduated cylinder. Tie a thread to the screw plug of the boat. Carefully lower the boat into the 100 ml of water. The boat will float with almost 80 ml of water beneath it. From this it can be seen that the boat would have floated in as little as 25 grams of water even though its weight is 150 grams.

While the boat is floating, suppose that the water level is 7.5 cm above the bottom of the boat. The hydrostatic pressure at the bottom of the boat is therefore 7.5 g/cm^2. The area of the bottom of the boat described is about 20 cm^2. The upward force on the bottom of the boat is therefore 150 g and equal to the weight of the floating boat.

The buoyant force on this boat depends only on the hydrostatic pressure at the level of the bottom of the boat, and on the area of the bottom of the boat. Like hydrostatic pressure, this force is independent of the total weight of the liquid.

The student sees that Archimedes' principle is just a consequence of hydrostatic pressure and is not a separate law of nature.

A Convenient Apparatus for the Diffraction Grating Experiment

G. P. Brewington
Lawrence Institute of Technology, Highland Park, Michigan

AVAILABILITY and convenience in storing apparatus are of prime importance in most college physics laboratories. The diffraction grating apparatus described below was developed for use in a laboratory room which is used during all terms of the sophomore physics course, both day and evening school. Rather interestingly, the apparatus in the hands of the average student gives results superior to those obtained heretofore.

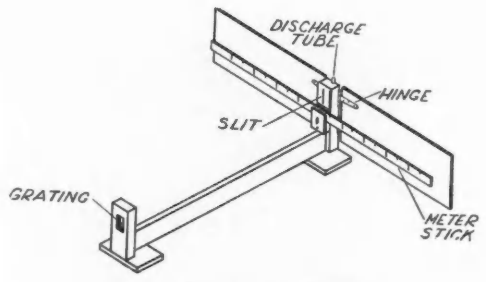

Fig. 1. Diffraction experiment apparatus.

The 15,000 lines/inch grating, referring to Fig. 1, is permanently mounted inside the upright with the glass face toward and as near the observer as possible. The replica surface on the rear then is well protected. At the other end of the apparatus, one meter distant, are mounted the slit, two wings, a clamp for fixing the meter stick, and the light source. The two wings (20 $\times$ 50 cm), hinged to the upright, are remarkably efficient in reducing extraneous images. A meter stick, held in place by a wing nut, may be removed and the wings folded into the apparatus thus considerably reducing the size for storage. A slit made of metal or black paper, mounted so as to be practically in the plane of the meter stick, reduces parallax to unobservable levels. This reduction of parallax was one of the unexpected advantages of the apparatus, for the location of the spectrum lines is completely independent of the location of the observer's eye. The light source may be a Geissler tube or even better, a short length (less than 6 in.) of commercial discharge tubing containing any of the available gases. Usually a local sign contractor will make these short lengths to specification. A special transformer known as a "clock" transformer (used to operate the circular fluorescent tubing commonly seen about clock faces) has been found to be very satisfactory for operating the commercial tubing. This transformer, which usually comes unmounted, develops about 2000 volts and will operate two or three of the discharge tubes connected in series. Two sets of the apparatus can thus be operated back-to-back on one transformer. The back-to-back arrangement has one further advantage in reducing electrical shock hazards. The entire apparatus is painted dull black. For this purpose a blackboard paint is recommended, since the abrasive present very materially increases the permanence of the paint.

Since the suggested light sources are sufficiently intense and since extraneous light in the field of view is reduced to a minimum the diffraction apparatus as described can be used in only slightly darkened rooms without interference from other experiments.

Letters to the Editor

The Acoustic Radiator

THE Schilling[1-2] articles of a decade ago gave impetus to the design of apparatus for the use of high frequency audible sound waves, in place of optical waves, to illustrate more easily Huyghen's wave principle, interference and diffraction phenomena.

One of the acoustical radiators,[3] manufactured to demonstrate the above principles, was purchased in 1939 by the University of Arkansas. The equipment has worked well except for the Galton whistle, which was used as the source of sound. Even with a constant pressure tank, the frequency would not always remain constant over any sequence of experiments. Following the war, the Galton whistle was replaced by a very small loud speaker placed within the tube. The sound was directed toward a parabolic reflector at the closed end. The reflected wave which passed through the open end has proved to be more satisfactory than that from the Galton whistle.

The main purpose of this letter is to call attention to the loud speaker system used in a more recent construction of a complete acoustical radiator at William Jewell College. In this instrument four small loud speakers, connected in series, were placed at A of Fig. 1, the location of the

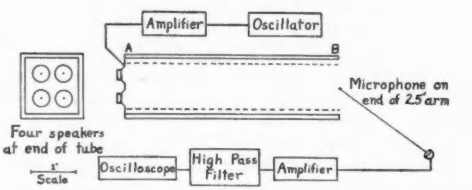

FIG. 1. An acoustic radiator designed to radiate plane waves at opening B by use of four speakers connected in series and located at end A.

parabolic reflector mentioned in the previous paragraph. This source has proved very satisfactory for a range of frequencies from 5000 to 10,000 cycle sec^{-1} to perform some 23 different experiments.

The tube is constructed of wood and lined inside with glass fiber one inch thick. The circular and rectangular slits are made from quarter-inch 3-ply wood. The semi-reflectors are made of painted screen wire. The arrangement for the oscillator, amplifiers, and oscilloscope is also shown in Fig. 1.

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L. B. HAM

University of Arkansas
Fayetteville, Arkansas

[1] H. K. Schilling and W. Whitson, *Am. Physics Teacher* **4**, 27 (1936).
[2] H. K. Schilling, *Am. Physics Teacher* **5**, 280 (1937); **6**, 156, 266 (1938).
[3] Central Scientific Co., Catalog J-141, Item No. 84685.

A Simple Acoustical Model of the Čerenkov Phenomenon

GIVEN a string of infinite length, along which sinusoidal waves of constant amplitude propagate continuously in one direction, we ask: What kind of waves will be generated by the string in the surrounding medium, e.g., in the open air?

The exact quantitative treatment of this question, considered as a hydrodynamical problem, is an extremely difficult task. If, however, one is satisfied with the purely qualitative description of the phenomena to be expected, this can be deduced by the simple application of the principle of Huyghens in its elementary form.

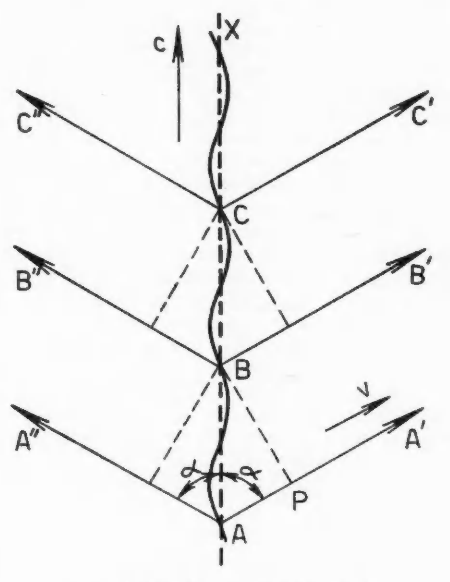

FIG. 1. Radiation from vibrating string.

Let AX be the vibrating string (Fig. 1), and let AA', BB', $CC'\cdots$ and also AA'', BB'', $CC''\cdots$ indicate directions, each enclosing an angle α with the string. Then, in accordance with the principle of Huyghens, vibrations of finite amplitude will be observable in this direction, provided that

$$AP = AB\cos\alpha = \lambda_s\cos\alpha = k\lambda_m, \tag{1}$$

where λ_s and λ_m denote the wavelength on the string and in the medium (air), respectively, and $k = 1, 2, 3\cdots$. Let c and v be the velocities of propagation on the string and in the medium, respectively. Then instead of Eq. (1) we can write

$$c\cos\alpha = kv, \tag{2}$$

and from this

$$\cos\alpha = kv/c, \tag{3}$$

where again $k=1, 2, 3\cdots$. Because $\cos\alpha\leqq 1$, Eq. (3) cannot be satisfied, if $v>c$, that is to say: *If the velocity of propagation in the surrounding medium is greater than along the string, no traveling wave will be generated by the latter*, or in other words: *no energy will be radiated from the string into the medium.* Although the medium in immediate proximity of the string will take part in the vibrations in this case also, the amplitudes of vibration will decrease exponentially with the distance from the string, exactly in the same way as those of the light waves penetrating into the second medium in the case of total reflection.

If, however, $v<c$, then Eq. (3) gives at least one real value for α, i.e., *in this case we observe sound waves emitted by the string, and propagating along the surface of a cone, coaxial with it and of aperture 2α.* Thus, our acoustical arrangement actually exhibits all characteristics of the Čerenkov radiation, Eq. (3) being identical with the equation determining the aperture of the latter.[1,2]

One particular circumstance should, however, be discussed and explained. In our model the vibrating string imparts a *periodic motion* to the medium, while in the Čerenkov effect the primary excitation, due to the impacting electrons, has the character of a *bremsstrahlung*, i.e., it consists of *aperiodic impulses*. Thus, we have to do with a process analogous to the excitation of white light and the function of the medium, in our model as well as in the Čerenkov effect, is similar to that of the prism of a spectroscope. Both the prism and the acoustical or optical medium perform a Fourier analysis; they resolve the aperiodic impulses into a continuous spectrum and arrange the different components, in virtue of their different refractivity, in space; the prism in accordance with the formula of minimal deviation, and the acoustical or optical medium in accordance with Eq. (3).

Finally, we should remark that if $v<c/2$, i.e., if the index of refraction of the optical medium, n, is greater than 2, and β, the ratio of the velocity of the electrons to that of light, is almost unity, then Eq. (3) or the equivalent equation of Čerenkov[1]

$$\cos\vartheta = 1/\beta n \tag{4}$$

has two solutions

$$\cos\vartheta_1 = 1/\beta n$$

and

$$\cos\vartheta_2 = 2/\beta n. \tag{5}$$

Thus, with such substances as the diamond ($n\approx 2.4$), cuprite ($n\approx 2.5$–2.9), calomel ($n\approx 2.6$), and furthermore, with some organic dyes, such as fuchsin ($n\approx 2.637$), malachite green ($n\approx 2.5$), Hoffmann violet ($n\approx 2.5$), the Čerenkov radiation must consist of *two* rings instead of *one*, the apertures of which are given by Eq. (5).

P. Selényi

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[1] P. A. Čerenkov, *Physical Rev.* **52**, 378 (1937).
[2] J. M. Frank and J. E. Tamm, *C. R. Acad. Sci. URSS* **14**, 109 (1937); see also George B. Collins and Victor G. Reiling, *Physical Rev.* **54**, 499 (1938).

Concerning Lecture-Demonstrations

It has long been glibly admitted that lecture-demonstrations are extremely useful teaching devices. Their pedagogical soundness appears to be substantially attested to by the provision some physics departments make for them. Very often vast money is put into demonstration equipment and special staff is available for setting up lectures. A picture is worth ten thousand words, we admit. The writer's experience along these lines dictates a little closer examination of this demonstration business and "an extension of the method" is proposed.

The *usual* lecture-demonstration procedure is well known. The experiment is set up on as large a scale as possible and with some drama (it is hoped the performer possesses it!) the routine is run through. (The language here is not idly chosen; drama is a necessary ingredient and without it the game is utterly lost to most of the observers.) We then run through the experiment and point out what has been proved. "Notice, ladies and gentlemen, that the theory we developed yesterday is here demonstrated. The predictions are borne out by the experiment." The bell rings, the class is dismissed, the equipment is put away, and the episode is closed. There may be some discussion among the better students as to what happened, and why, but it is safe to say that except by these few little of the experiment was understood.

For several years now the writer has resorted to a scheme which has shown itself to be highly sound instructionally. *The student is required to write up the demonstration experiment in a bound notebook reserved for these write-ups alone.* A title is suggested but the student is free to name it as he pleases. The usual procedure is to write, in one's own words, with reference to diagrams, a reasonably thorough statement of what was done and what happened, and *why.* Some enterprising souls do the job astonishingly well. Some give a brief historical comment, if one is appropriate; some carry the experiment to interesting extensions. *But the majority of the write-ups are regrettably inadequate. It is easily discovered that most of the people did not know what was going on.* In addition, it is verified that their command of the written word, let alone the physics, is abysmally dim. But this last is common knowledge.

The reports are read critically by the teacher. This takes time, of course, and on this count alone the method will not find much favor. But it is highly informative for the instructor to find that not much of what he did or said took root. On the strength of this he can improve his demonstrations. These lecture-demonstration reports are required of the student *in addition to* the regular laboratory reports.

The usefulness to the student is obvious. He is under some compulsion to watch the demonstration with sharper attention and some *critical* thinking is forced upon him outside the classroom. Of this last there is much too little.

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Author Index to Volume 17

In this index are listed the names of authors and the titles of their articles and notes. Abstracts of papers read at meetings and of articles appearing under "Digest of Periodical Literature" in the Journal are generally not listed; however, they are indexed in the Analytic Subject Index. Those which are included in the Author Index are designated by the symbol (D). The symbol (L) after an entry signifies a Letter to the Editor.

Analytic Subject Index to Volume 17

The titles of articles are disregarded, the entries being based on analyses of the contents of the articles. The symbol (D) designates a digest of an article or an abstract of a paper read at a meeting: (T) designates title only and (L) designates a Letter to the Editor.

To facilitate reference to any desired subject, the index is divided into sections arranged alphabetically. The titles of these sections and of cross references to them are as follows:

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